STANDARDS OF LEARNING
CONTENT REVIEW NOTES

ALGEBRA II

4th Nine Weeks, 2018-2019

SUFFOLK PUBLIC SCHOOLS
Algebra II Content Review Notes are designed by the High School Mathematics Steering Committee as a resource for students and parents. Each nine weeks’ Standards of Learning (SOLs) have been identified and a detailed explanation of the specific SOL is provided. Specific notes have also been included in this document to assist students in understanding the concepts. Sample problems allow the students to see step-by-step models for solving various types of problems. A “TRY IT” section has also been developed to provide students with the opportunity to solve similar problems and check their answers. Supplemental online information can be accessed by scanning QR codes throughout the document. These will take students to video tutorials and online resources. In addition, a self-assessment is available at the end of the document to allow students to check their readiness for the nine-weeks test.

The document is a compilation of information found in the Virginia Department of Education (VDOE) Curriculum Framework, Enhanced Scope and Sequence, and Released Test items. In addition to VDOE information, Prentice Hall Textbook Series and resources have been used. Finally, information from various websites is included. The websites are listed with the information as it appears in the document.

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The Algebra II Blueprint Summary Table is listed below as a snapshot of the reporting categories, the number of questions per reporting category, and the corresponding SOLs.

### Algebra II

#### Test Blueprint Summary Table

<table>
<thead>
<tr>
<th>Reporting Category</th>
<th>Algebra II SOL</th>
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*Field-test items are being tried out with students for potential use on subsequent tests and will not be used to compute students’ scores on the test.*
Geometric Formulas:

\[ A = \frac{1}{2}bh \quad \text{Area of a triangle} \]

\[ p = 2l + 2w \quad \text{Perimeter of a rectangle} \]

\[ A = lw \quad \text{Area of a rectangle} \]

\[ a^2 + b^2 = c^2 \quad \text{Pythagorean theorem} \]

Quadratic Formula:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

where \( ax^2 + bx + c = 0 \) and \( a \neq 0 \)

Statistics Formulas:

Given:

- \( x \) represents an element of the data set,
- \( x_i \) represents the \( i^{th} \) element of the data set,
- \( n \) represents the number of elements in the data set,
- \( \mu \) represents the mean of the data set,
- \( \sigma \) represents the standard deviation of the data set, and
- \( \sigma^2 \) represents the variance of the data set

**z-score:**

\[ z = \frac{x - \mu}{\sigma} \]

**Standard deviation:**

\[ \sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}} \]

**Variance:**

\[ \sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n} \]
Sequence and Series Formulas:

Given:

\( a_n \) represents the value of \( n^{th} \) term
\( S_n \) represents the sum of first \( n \) terms
\( S_\infty \) represents the sum of an infinite geometric series
\( r \) represents the common ratio
\( d \) represents the common difference

**Arithmetic**

\[
a_n = a_1 + (n - 1)d
\]

\[
a_n = a_{n-1} + d
\]

\[
S_n = \frac{n}{2}(a_1 + a_n)
\]

\[
S_n = \frac{n}{2}[2a_1 + (n - 1)d]
\]

**Geometric**

\[
a_n = a_1r^{n-1}
\]

\[
a_n = a_{n-1} \cdot r
\]

\[
S_n = \frac{a_1(1-r^n)}{(1-r)}, \ r \neq 1
\]

\[
S_\infty = \frac{a_1}{(1-r)}, \ |r| < 1
\]

Permutations and Combinations Formulas:

If \( n \) and \( r \) are positive integers and \( n \geq r \),

\[
\begin{align*}
\text{Permutations} & : \\
\quad \quad n^P_r & = \frac{n!}{(n-r)!} \\
\text{Combinations} & : \\
\quad \quad n^C_r & = \frac{n!}{r!(n-r)!}
\end{align*}
\]
## Standard Normal Probabilities

Table entry for $z$ is the area under the standard normal curve to the left of $z$.

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Sequences and Series

AII.5 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve practical problems, including writing the first \( n \) terms, determining the \( n^{th} \) term, and evaluating summation formulas. Notation will include \( \sum \) and \( a_n \).

<table>
<thead>
<tr>
<th>Arithmetic Sequence</th>
<th>Geometric Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>A sequence where the difference between consecutive terms is a constant. (You add or subtract a constant value)</td>
<td>A sequence where the difference between consecutive terms is a common ratio. (You multiply or divide a constant value)</td>
</tr>
<tr>
<td>Examples: 3, 5, 7, 9, 11… (constant is +2) 25, 20, 15, 10… (constant is −5)</td>
<td>Examples: 3, 6, 12, 24, 48… (ratio is ( \frac{2}{1} )) 6, 9, 13.5, 20.25, 30.375… (ratio is ( \frac{3}{2} ))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Formula</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_n = a + (n - 1)d )</td>
<td>( a_n = a \cdot r^{n-1} )</td>
</tr>
<tr>
<td>( a ) is the starting value, ( d ) is the common difference and ( n ) is the number of terms.</td>
<td>( a ) is the starting value, ( r ) is the common ratio and ( n ) is the number of terms.</td>
</tr>
</tbody>
</table>

Example 1: What is the 35th term of the arithmetic sequence that begins 7, 4…

\[
a_n = a + (n - 1)d
\]

Substitute your values \((a = 7, n = 35, d = -3)\)

\[
a_n = 7 + (35 - 1)(-3)
\]

Simplify

\[
a_n = -95
\]

Example 2: What is the 20th term of the geometric sequence that begins 1, 2, 4…

\[
a_n = a \cdot r^{n-1}
\]

Substitute your values \((a = 1, r = 2, n = 20)\)

\[
a_n = 1 \cdot 2^{20-1}
\]

Simplify

\[
a_n = 524,288
\]

Example 3: What is the missing term in this geometric sequence 9, □, 1…

\[
a_n = a \cdot r^{n-1}
\]

Substitute your values \((a_n = 1, a = 9, n = 3)\)

\[
1 = 9 \cdot r^{3-1}
\]

Simplify

\[
\frac{1}{9} = r^2
\]

Solve for the common ratio, \( r \).

\[
r = \frac{1}{3}
\]

The missing term is \( 9 \cdot \frac{1}{3} = 3 \)
A series is the sum of a geometric or arithmetic sequence.

| Sum of a Finite Arithmetic Series | Sum of a Finite Geometric Series | Sum of an Infinite Geometric Sequence (Only applicable for $|r| < 1$) |
|----------------------------------|---------------------------------|-------------------------------------------------|
| $S_n = \frac{n}{2} \left( a_1 + a_n \right)$ | $S_n = \frac{a_1 (1 - r^n)}{1 - r}$ | $S_n = \frac{a_1}{1 - r}$ |
| Where $a_1$ is the first term, $a_n$ is the $n^{th}$ term, and $n$ is the number of terms. | Where $a_1$ is the first term, $r$ is the common ratio, and $n$ is the number of terms. | . Where $a_1$ is the first term, and $r$ is the common ratio. |

You may see series written in Summation Notation

You can write the series 7+9+11+ …+89 as

$$\sum_{n=1}^{42} (2n + 5)$$

**Example 4:** Evaluate

Because the explicit formula is linear, this will be an arithmetic series. In order to evaluate an arithmetic series we need to know the first and last term and number of terms.

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$S_n = \frac{42}{2} (7 + 89)$$

$$S_n = 2016$$

Substitute your values

$(n = 42, a_1 = 2(1) + 5 = 7, a_{42} = 2(42) + 5 = 89)$

Simplify

**Try It:** Sequences and Series

1. What is the 9th term of the geometric sequence that begins 2, 1, …
2. What is the missing term in this arithmetic sequence 12, □, 25 ...
3. Write the series in summation notation 120 + 115 + 110 + 105 + 100 + 95
4. Find the sum of the geometric series

$$\sum_{n=1}^{10} 4^n$$

Scan this QR code to go to a video tutorial on sequences and series.
### Statistics

**All.12 The student will compute and distinguish between permutations and combinations.**

You can use multiplication to quickly determine the number of ways a certain event can happen. You can use Permutations and Combinations to determine the number of ways. You use Combinations when order is not important and Permutations when the order is important. These formulas are on your formula sheet, but your formula sheet does not tell you when to use each formula.

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n^P_r = \frac{n!}{(n-r)!}$</td>
<td>Permutation</td>
<td>$8^P_2 = \frac{8!}{(8-2)!} = \frac{8!}{6!} = \frac{40320}{720} = 56$</td>
</tr>
<tr>
<td>$n^C_r = \frac{n!}{r!(n-r)!}$</td>
<td>Combination</td>
<td>$8^C_2 = \frac{8!}{2!(8-2)!} = \frac{8!}{2!(6!)} = \frac{40320}{1440} = 28$</td>
</tr>
</tbody>
</table>

**Example 1:** In a diving meet, the top 8 divers advance to the finals and scores are cleared. In how many ways can a field of 14 divers qualify to the finals?

Because all of the top 8 advance to the finals, the order that they finish isn’t important as long as they are in the top 8. This means we will use a combination!

$$14^C_8 = \frac{14!}{8!(14-8)!} = \frac{14!}{8!(6!)} = \frac{8.71782912 \times 10^{10}}{29030400} = 3,003$$

There are 3,003 possible combinations of divers that can advance to the finals.
Example 2: In the finals of the diving meet referenced in Example 1, the top 3 finishers score points for their team. First place receives 10 points, 2\textsuperscript{nd} place receives 8 points, and 3\textsuperscript{rd} place receives 6 points. In how many ways can the 8 finalists finish in the top 3?

Now, the order is important because 1\textsuperscript{st} place gets more points than 2\textsuperscript{nd} place. We will use a permutation!

\[ 8P_3 = \frac{8!}{(8 - 3)!} = \frac{8!}{5!} = \frac{40320}{120} = 336 \]

There are 336 possible ways that the top 8 divers can finish in the top 3.

TRY IT: Statistics

1. A teacher is making a multiple choice quiz. She wants to give each student the same questions, but have each student's questions appear in a different order. If there are twenty-seven students in the class, what is the least number of questions the quiz must contain?

2. A coach must choose five starters from a team of 12 players. How many different ways can the coach choose the starters?

Standard Deviation

All.11 The student will

a) identify and describe properties of a normal distribution;
b) interpret and compare z-scores for normally distributed data; and
c) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.

Standard Deviation and Variance

The standard deviation of a data set tells us how “spread out” the data is, if the data is very spread out, the standard deviation will be higher than if the data is all clumped together. The variance is another measure of how spread out the data is.

Standard deviation is represented by \( \sigma \) (lowercase Greek letter sigma). The variance is just the standard deviation squared, \( \sigma^2 \).

There is a way to calculate these values in the graphing calculator.
Example 1: The height in inches of the Washington Wizards starting lineup is shown below. Find the standard deviation and the variance of the data, round your answer to the nearest hundredth.

75, 80, 76, 79, 81

Start by entering the data into L1 in your STAT menu.

Then go to STAT, scroll over to CALC, and select 1: 1-Var Stats

When you press ENTER twice, your calculator will display the single variable statistics.

We want to use the standard deviation that is represented by $\sigma$, therefore our standard deviation is 2.32. The variance is just the standard deviation squared: $\sigma^2 = (2.32)^2 = 5.38$

Standard Deviation = 2.32 inches
Variance = 5.38 inches

Example 2: Using the data from Example 1, how many of the starting lineups’ heights are within one standard deviation of the mean?

The heights were 75, 80, 76, 79, 81

This question is referring to players who are both one standard deviation above the mean and one standard deviation below the mean.

The mean was 78.2 inches, and the standard deviation was 2.32 inches.

$78.2 + 2.32 = 80.52$ inches

$78.2 - 2.32 = 75.88$ inches
There is one player (81") who is taller than one standard deviation above the mean and one player (75") who is shorter than one standard deviation below the mean. This means that 3 players (80", 76", and 79") are all within one standard deviation of the mean.

**Example 3:** How short would a player have to be in order to be 2.5 standard deviations below the mean?

First we need to calculate how many inches is 2.5 standard deviations. We can do this by multiplying the standard deviation by 2.5.  

\[ 2.32 \cdot 2.5 = 5.8 \text{ inches} \]

We can then subtract 5.8" from the mean of 78.2".  

\[ 78.2 - 5.8 = 72.4 \text{ inches} \]

A player would have to be 72.4" tall to be 2.5 standard deviations below the mean.

**TRY IT:**  **Standard Deviation**  
Use the speeds of the top 10 fastest roller coasters, provided in mph, to answer the questions below.

128, 120, 107, 100, 100, 95, 93, 85, 85, 82

1. What is the standard deviation? (round to nearest hundredth)
2. What is the variance? (round to nearest hundredth)
3. How many coasters are within 1.25 standard deviations of the mean?
4. How fast would a coaster have to be going to be 3 standard deviations above the mean?

**Z-Scores**

A z-score tells us how many standard deviations a specific data point is from the mean. Z-scores can be positive or negative. If a z-score is positive it indicates that the data point is that many standard deviations above the mean, if a z-score is negative it indicates that the data point is that many standard deviations below the mean.

If a data point has a z-score of zero, then that data point is the same as the mean of the data.
The formula for calculating a z-score is on the Algebra II formula sheet.

\[
\text{z-score (z)} = \frac{x - \mu}{\sigma}
\]

\[
z - \text{score} = \frac{\text{Data Point} - \text{MEAN}}{\text{Standard Deviation}}
\]

**Example 4:** A class’s history midterm grades are shown below. What is the z-score for a score of 78?
Grades: 81, 62, 90, 77, 82, 86, 98, 100, 90, 75, 83, 88, 79, 76, 85

First calculate the 1-Var Stats in your calculator. We need the Mean and Standard Deviation.

<table>
<thead>
<tr>
<th>1-Var Stats</th>
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</thead>
<tbody>
<tr>
<td>(x = 83.47)</td>
</tr>
<tr>
<td>(\sum x = 1252)</td>
</tr>
<tr>
<td>(\sum x^2 = 105758)</td>
</tr>
<tr>
<td>(s_x = 9.16)</td>
</tr>
<tr>
<td>(n = 15)</td>
</tr>
</tbody>
</table>

We want to know the z-score for a 78.

\[
z - \text{score} = \frac{78 - 83.47}{9.16} = -0.597
\]

This shows that a score of 78 is a little more than \(\frac{1}{2}\) standard deviation below the mean.

**Example 5:** Jessie’s teacher wouldn’t tell her the actual score that she received; only that she had a z-score of 1.26. Determine Jessie’s score.

Now we are given the z-score and asked to find the data point. Plug everything that you know into the formula, and then solve the equation for the missing piece.

\[
z - \text{score} = \frac{\text{Data Point} - \text{MEAN}}{\text{Standard Deviation}}
\]

We know the z-score, mean and standard deviation.

Now we just need to solve for \(X\)!

\[
1.26 = \frac{X - 83.47}{9.16}
\]

\[
X = 11.542 \times 9.16 + 83.47 + 83.47
\]

\[
95.01 = X
\]
Example 6: Another class took the exam and had the same class average but a standard deviation of 3.60. If Jessie had been in this class (and still had a z-score of 1.26), would her score be lower or higher? Explain.

We could calculate the answer to this problem, but this isn’t necessary. The class average is exactly the same, but the standard deviation is lower. Jessie still scored 1.26 standard deviations above the mean, but now those standard deviations are smaller, therefore her grade will be lower.

\[
z \text{- score} = \frac{\text{Data Point} - \text{MEAN}}{\text{Standard Deviation}}
\]

We know the z-score, mean and standard deviation.

\[
1.26 \cdot 3.60 = X - 83.47
\]

Now we just need to solve for X!

\[
4.536 = X - 83.47
\]

\[
+83.47 + 83.47
\]

\[
88.01 = X
\]

Jessie’s score was an 88%

Normal Curve

In a normal distribution:
- 68% of the data will fall within 1 standard deviation of the mean
- 95% of the data will fall within 2 standard deviations of the mean
- 99.7% of the data will fall within 3 standard deviations of the mean
Example 7: The graph below shows how temperatures were normally distributed across the globe one day last year. If 1,000 cities were sampled, how many cities had temperatures between 4° and 100°?

4° is 2 standard deviations below the mean and 100° is 2 standard deviations above the mean. 95% of the data points fall within 2 standard deviations of the mean, therefore 95% of the cities sampled would have had temperatures between 4° and 100°.

\[ 0.95 \cdot 1000 = 950 \text{ cities} \]

Z-Tables

When z-scores fall on an integer, it is easy to use the 68-95-99.7 rule to determine the cumulative probability of a range of values. Sometimes, you may need to use a z-table to determine the probability based on a z-score when the z-score is in decimal form.

In order to use a z-table, follow the column on the left to find your ones and tenths place of your z-score. Then, follow the row at the top to find your hundredths place of your z-score. Find where they meet. This value represents the cumulative probability or area under the curve to the left of the z-score.
Example 1:
Use the z-table to determine the probability that a data value will fall below a data value associated with a z-score of 0.53.

Look at what the value represents. Remember, that this value represents area to the left of the curve! If you were asked to find probability of values “greater than” the value associated with 0.53, you would have to subtract the value from 1.

\[ 1 - 0.7019 = 0.2981 \]

Example 2:
The length of the life of an instrument is 12 years with a standard deviation of 4 years. Out of 500 instruments, how many can be expected to last 15 or more years?

First, the z-score of a length of 15 must be found.

\[
\text{z-score} = \frac{\text{Data Point} - \text{MEAN}}{\text{Standard Deviation}} = \frac{15 - 12}{4} = 0.75
\]

Next, use the z-table above to find the probability value of a z-score of 0.75. You should find 0.7734 (or 77.34%).

Remember, subtract this value from 1, because we are looking for the area under the curve to the right since we were asked about 15 or more years!

\[ 1 - 0.7734 = 0.2266 \]

Multiply 0.2266 by 500. \[0.2266 \cdot 500 = 113.3 \approx 113\]

113 instruments represents 22.66% of 500 instruments.
TRY IT:  **Standard Deviation**

The heights of the tallest 7 men ever recorded are shown below (in inches). Use these to answer the questions.

107, 105, 103.5, 99, 99, 99, 98

5. What is the z-score for 99 inches?
6. What is the z-score for 107 inches?
7. The tallest woman ever confirmed would have had a z-score of -1.13. How tall was she?
8. The means and standard deviations for two schools’ SAT scores is shown below.
   The z-score for the 95th percentile is 1.598. By how many points do the 95th percentile scores differ for each school? (Round to the nearest whole number.)
   School A: Mean = 1520, Standard Deviation = 110
   School B: Mean = 1490, Standard Deviation = 155
9. The height of the men in the United States is normally distributed as shown in the graph. The mean is 69.25” with a standard deviation of 2.5”. What percent of the heights are between 66.75” and 74.25”?

10. The length of time that people can hold their breath under water is normally distributed with a mean of 32 seconds and a standard deviation of 12 seconds. Out of 750 people, about how many people would be expected to hold their breath for 42 seconds or longer? For 35 seconds or less? (Hint: Use the part of the z-table shown on page 15.)
Answers to the **TRY IT** problems:

**Sequences and Series**

1. \(\frac{1}{128}\)
2. 18.5
3. \(\sum_{n=1}^{6}(-5n + 125)\)
4. 1,398,100

**Statistics**

1. 5 questions would give 120 order possibilities
2. 792

**Standard Deviation**

1. 14.42 mph
2. 207.94
3. 8
4. 142.76 mph
5. −0.75
6. 1.65
7. 97.74 inches
8. 42 points
9. 81.5%
10. About 152 people can hold their breath for 42 seconds or longer; about 449 people can hold their breath for 35 seconds or less.