Algebra II Content Review Notes are designed by the High School Mathematics Steering Committee as a resource for students and parents. Each nine weeks’ Standards of Learning (SOLs) have been identified and a detailed explanation of the specific SOL is provided. Specific notes have also been included in this document to assist students in understanding the concepts. Sample problems allow the students to see step-by-step models for solving various types of problems. A “TRY IT” section has also been developed to provide students with the opportunity to solve similar problems and check their answers. Supplemental online information can be accessed by scanning QR codes throughout the document. These will take students to video tutorials and online resources. In addition, a self-assessment is available at the end of the document to allow students to check their readiness for the nine-weeks test.

The document is a compilation of information found in the Virginia Department of Education (VDOE) Curriculum Framework, Enhanced Scope and Sequence, and Released Test items. In addition to VDOE information, Prentice Hall Textbook Series and resources have been used. Finally, information from various websites is included. The websites are listed with the information as it appears in the document.

Supplemental online information can be accessed by scanning QR codes throughout the document. These will take students to video tutorials and online resources. In addition, a self-assessment is available at the end of the document to allow students to check their readiness for the nine-weeks test.

To access the database of online resources scan this QR code, or visit http://spsmath.weebly.com

The Algebra II Blueprint Summary Table is listed below as a snapshot of the reporting categories, the number of questions per reporting category, and the corresponding SOLs.

<table>
<thead>
<tr>
<th>Reporting Category</th>
<th>Algebra II SOL</th>
<th>Number of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expressions and Operations</td>
<td>AII.1a-c</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>AII.2</td>
<td></td>
</tr>
<tr>
<td>Equations and Inequalities</td>
<td>AII.3a-d</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>AII.4</td>
<td></td>
</tr>
<tr>
<td>Functions and Statistics</td>
<td>AII.5</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>AII.6a-b</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AII.7a-k</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AII.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AII.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AII.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AII.11a-c</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AII.12</td>
<td></td>
</tr>
<tr>
<td>Number of Operational Items</td>
<td></td>
<td>45</td>
</tr>
<tr>
<td>Number of Field Test Items*</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Total Number of Items on Test</td>
<td></td>
<td>55</td>
</tr>
</tbody>
</table>

*Field-test items are being tried out with students for potential use on subsequent tests and will not be used to compute students’ scores on the test.

It is the Mathematics Instructors’ desire that students and parents will use this document as a tool toward the students’ success on the end-of-year assessment.
Algebra II Formula Sheet
2009 Mathematics Standards of Learning

Geometric Formulas:

\[ A = \frac{1}{2}bh \]
\[ p = 4s \]
\[ p = 2l + 2w \]
\[ a^2 + b^2 = c^2 \]

Quadratic Formula:
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } ax^2 + bx + c = 0 \text{ and } a \neq 0 \]

Statistics Formula:
Given:
\( x \) represents an element of the data set,
\( \mu \) represents the mean of the data set, and
\( \sigma \) represents the standard deviation of the data set.
\[ z \text{-score} (z) = \frac{x - \mu}{\sigma} \]

Permutations and Combinations Formulas:
If \( n \) and \( r \) are positive integers and \( n \geq r \),
\[ n^P_r = \frac{n!}{(n - r)!} \]
\[ n^C_r = \frac{n!}{r!(n - r)!} \]

Sequence and Series Formulas:
Given:
\( a_n \) represents the value of \( n^{th} \) term
\( S_n \) represents the sum of first \( n \) terms
\( S_\infty \) represents the sum of an infinite geometric series
\( r \) represents the common ratio
\( d \) represents the common difference

Arithmetic
\[ a_n = a_1 + (n - 1)d \]
\[ S_n = \frac{n}{2}(a_1 + a_n) \]

Geometric
\[ a_n = a_1 r^{n-1} \]
\[ S_n = \frac{a_1(1 - r^n)}{1 - r}, \quad r \neq 1 \]
\[ S_\infty = \frac{a_1}{1 - r}, \quad |r| < 1 \]

VDOE has not released final version of Spring 2019 formula sheet.
Linear Equations and Inequalities

A.4 The student will solve multistep linear and quadratic equations in two variables,
  b) justifying steps used in simplifying expressions and solving equations, using field
    properties and axioms of equality that are valid for the set of real numbers and its subsets;
  d) solving multistep linear equations algebraically and graphically;
A.5 The student will solve multistep linear inequalities in two variables, including
  a) solving multistep linear inequalities algebraically and graphically;

You will solve an **equation** to find all of the possible values for the variable. In order to solve an equation you will need to **isolate the variable** by performing **inverse operations** (or ‘undoing’ what is done to the variable).

Any operation that you perform on one side of the equal sign **MUST** be performed on the other side as well. Drawing an arrow down from the equal sign may help remind you to do this.

**Example 1:**

\[
\frac{x+4}{5} = -12
\]

\[
\cdot 5 \quad \cdot 5
\]

\[
x + 4 = -60
\]

\[
-4 \quad - 4
\]

\[
x = -64
\]

Check your work by plugging your answer back in to the original problem.

\[
\frac{-64 + 4}{5} = \frac{-60}{5} = -12 \quad \checkmark
\]

You may be asked to write your answer in **set notation**. In this example, the only element that would be included in the set is the number -64. You could write this solution in set notation as:

\[
\{-64\} \text{ or } \{x \mid x = -64\} \text{ or } \{x : x = -64\}
\]

This is read as the set of all \( x \), such that \( x \) is equal to negative sixty four.

Sets can include more than one element, such as the set of all high schools in Suffolk:

\[
\{KFHS, LHS, NRHS\} \text{ or } \{x \mid x \text{ is a high school in Suffolk}\}
\]

\[
\quad \text{or } \{x : x \text{ is a high school in Suffolk}\}
\]

Or infinitely many elements, such as the set of all positive whole numbers:

\[
\{1, 2, 3, 4, \ldots \infty\} \text{ or } \{x \mid x \text{ is a positive whole number}\}
\]

\[
\text{or } \{x : x \text{ is a positive whole number}\}
\]
You may have to distribute a constant, and combine like terms before solving an equation.

If there are variables on both sides of the equation, you will need to move them all to the same side in the same way that you move numbers.

Example 2:

\[
\begin{align*}
3p - 5 &= 7(p - 3) \\
3p - 5 &= 7p - 21 \\
-3p &= -3p \\
-5 &= 4p - 21 \\
+21 &= +21 \\
16 &= 4p \\
\div 4 &= \div 4 \\
p &= 4
\end{align*}
\]

\{4\} or \{p \mid p = 4\} or \{p : p = 4\}

Check your work by plugging your answer back in to the original problem.

\[
\begin{align*}
3(4) - 5 &= 7(4) - 21 \\
12 - 5 &= 28 - 21 \\
7 &= 7
\end{align*}
\]

Inequalities

An inequality is solved the same way as an equation. The only important thing to remember is that if you multiply or divide by a negative number, you need to switch the direction of the inequality sign. A proof of this is included in the online video tutorial or on the bottom of page 34 in your text book.

You will also need to know how to graph inequalities on the number line. If the inequality has a greater than or equal to (≥) or less than or equal to (≤) sign, then you will use a closed point to mark the spot on the number line. This closed point indicates that the number that the point is on IS included in the solution. For a greater than (>) or less than (<) sign, you will use an open point on the number line. This open point indicates that the number that the point is on is NOT included in the solution.
Example 1: Solve and graph the following inequality.

\[ y + 3 < -4 \]

\[ -3 \]

\[ y < -7 \]

Graph:

This example says that \( y \) is less than -7. In set notation: \( \{ y | y < -7 \} \) or \( \{ y : y < -7 \} \)

Another way this could be written is in interval notation. In this notation, an interval is represented as a pair of numbers. Parenthesis and/or brackets are used to indicate if the endpoints are included or excluded (like the open and closed dots when graphing inequalities on the number line).

The solution to Example 1 in interval notation is \( (-\infty, -7) \), which says all real numbers from negative infinity up to (but not including!) -7.

Example 2: \( 7m + 4 \geq 3m - 16 \)

\[ -3m \]

\[ 4m + 4 \geq -16 \]

\[ -4 \]

\[ 4m \geq -20 \]

\[ \div 4 \]

\[ m \geq -5 \]

\( \{ m | m \geq -5 \} \) or \( [-5, \infty) \)
Example 3: Ava’s math quiz scores are 78, 61, 80, and 65. What is the minimum score she would need on her 5th quiz to have a quiz average of at least 70?

\[
\frac{78+61+80+65+x}{5} \geq 70 \\
284+x \geq 70 \\
\cdot 5 \quad \cdot 5 \\
284 + x \geq 350 \\
-284 -284 \\
x \geq 66
\]

\[ \{x \mid x \geq 66 \} \text{ or } [66, \infty) \]

*If the highest score possible is 100, then you could write the answer* [66, 100]

Ava needs to score between 66 and 100 on her final quiz to have a 70% quiz average.

**Try It:**  *Linear Equations and Inequalities*

1. \[-17 = \frac{y-6}{2} \]
2. \[5(2n + 6) + 8 = 33 \]
3. \[3 - (4k + 2) = -15 \]
4. \[5g + 4 = -9g - 10 \]
5. \[-2(-4m - 1) + 3m = 4m - 8 + m \]
6. \[\frac{x-4}{2} = \frac{-2(3x-3)}{6} \]
7. Solve and graph: \[3x + 3 > 18 \]
8. Solve and graph: \[9w + 5 - 11w \geq -11 \]
9. Solve: \[2 - (6g - 8) < 4 (g - 5) \]
10. Write in interval notation: \[-9 < x \leq 10 \]
11. Write in interval notation: In order to win baseball tickets, Jamie must sell at least 20, but not more than 45 raffle tickets.
12. A salesman earns $410 per week plus 10% commission on sales. How many dollars in sales will the salesman need in order to make more than $600 for the week?
### Properties of Real Numbers

**A.4** The student will solve multistep linear and quadratic equations in two variables, b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets;

**AII.3** The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of i, and identify field properties that are valid for the complex numbers.

<table>
<thead>
<tr>
<th>Property</th>
<th>Definition</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplicative Property of Zero</td>
<td>Any number multiplied by zero always equals zero.</td>
<td>$a(0) = 0$  \hspace{1em}  $0 \cdot (-14) = 0$</td>
</tr>
<tr>
<td>Additive Identity</td>
<td>Any number plus zero is equal to the original number.</td>
<td>$a + 0 = a$  \hspace{1em}  $126 + 0 = 126$</td>
</tr>
<tr>
<td>Multiplicative Identity</td>
<td>Any number times one is the original number.</td>
<td>$a \cdot 1 = a$  \hspace{1em}  $1 \cdot 78 = 78$</td>
</tr>
<tr>
<td>Additive Inverse</td>
<td>A number plus its opposite always equals zero.</td>
<td>$a + (-a) = 0$  \hspace{1em}  $-21 + 21 = 0$</td>
</tr>
<tr>
<td>Multiplicative Inverse</td>
<td>A number times its inverse (reciprocal) is always equal to one.</td>
<td>$a \cdot \frac{1}{a} = 1$  \hspace{1em}  $\frac{5}{2} \cdot \frac{2}{5} = 1$</td>
</tr>
<tr>
<td>Associative Property</td>
<td>When adding or multiplying numbers, the way that they are grouped does not affect the outcome.</td>
<td>$(a + b) + c = a + (b + c)$  \hspace{1em}  $5 + (3 + 8) = (5 + 3) + 8$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(ab)c = a(bc)$  \hspace{1em}  $6(3a) = (6 \cdot 3) a$</td>
</tr>
<tr>
<td>Commutative Property</td>
<td>The order that you add or multiply numbers does not change the outcome.</td>
<td>$a + b = b + a$  \hspace{1em}  $14 + 6 = 6 + 14$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$ab = ba$  \hspace{1em}  $8 \cdot 3 = 3 \cdot 8$</td>
</tr>
<tr>
<td>Distributive Property</td>
<td>For any numbers a, b, and c: $a(b + c) = ab + ac$</td>
<td>$5(3 - 2) = 5 \cdot 3 - 5 \cdot 2$  \hspace{1em}  $-3(a + b) = -3a + (-3b)$  \hspace{1em}  $or \ -3(a + b) = -3a - 3b$</td>
</tr>
<tr>
<td>Substitution property of</td>
<td>If a = b then b can replace a.</td>
<td>If $5 + 2 = 7$, then $(5 + 2) \cdot 4 = 7 \cdot 4$</td>
</tr>
</tbody>
</table>
### Equality

<table>
<thead>
<tr>
<th>Equality</th>
<th>A quantity may be substituted for its equal in any expression.</th>
<th>If ( a = 5 ), then ( 11a = 11 \cdot 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive Property of Equality</td>
<td>Any quantity is equal to itself.</td>
<td>( a = a ) ( \frac{5}{5} = \frac{3}{3} )</td>
</tr>
</tbody>
</table>
| Transitive Property of Equality | If one quantity equals a second quantity and the second quantity equals a third, then the first equals the third. | \( \text{If } a = b, \text{and } b = c, \text{then } a = c. \)
\( \text{If } 2 + 4 = 6, \text{and } 2(3) = 6, \text{then } 2 + 4 = 2(3) \) |
| Symmetric Property of Equality | If one quantity equals a second quantity, then the second quantity equals the first. | \( \text{If } a = b, \text{then } b = a \)
\( \text{If } 25 = 13a - 1, \text{then } 13a - 1 = 25 \) |

### The Complex Number System

The **natural numbers** are the numbers you count (i.e. **counting numbers**). The **whole numbers** are the natural numbers and the number zero. The **integers** are all of the positive and negative whole numbers. **Rational numbers** are numbers that can be expressed as a quotient of two integers. These include all decimals that terminate or repeat. **Irrational numbers** have decimal representations that never terminate or repeat.
TRY IT: Properties of Real Numbers
Match the example on the left to the appropriate property on the right.

1. \((x + 3) + y = x + (3 + y)\)  
   A. Multiplicative Property of Zero
2. \(1 = \frac{1}{2x} \cdot 2x\)  
   B. Additive Identity
3. \(3x + 6 = 6 + 3x\)  
   C. Multiplicative Identity
4. \((5 - x)2 = 10 - 2x\)  
   D. Additive Inverse
5. \(b^5 + 0 = b^5\)  
   E. Multiplicative Inverse
6. \((x + 3) + y = y + (x + 3)\)  
   F. Associative Property
7. \(0 \cdot 17n = 0\)  
   G. Commutative Property
8. \(5x + (-5x) = 0\)  
   H. Distributive Property
9. \(xyz = xyz\)  
   I. Substitution Property of Equality
10. If one dollar is the same as four quarters, and four quarters is the same as ten dimes, then ten dimes is the same as one dollar.
   J. Reflexive Property of Equality
   K. Transitive Property of Equality
   L. Symmetric Property of Equality

Linear Functions

A.6 The student will graph linear equations and linear inequalities in two variables, including 
a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; and
b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.

A.7 The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically.

The domain is the set of all inputs, or x-coordinates in a relation.
The range is the set of all outputs, or y-coordinates in a relation.

In order for a relation to be classified as a function, each element in the domain can be paired with only one element in the range (i.e. the x-coordinates cannot repeat).
Example 1: Identify the domain and range, and state if the relation is a function.

\[\{(3, -1), (1, 0), (-2, -1), (-3, 4)\}\]

Domain: \{3, 1, -2, -3\}  
Range: \{-1, 0, 4\}

Because no value in the domain repeats, this relation IS a function.

Example 2: Identify the domain and range, and state if the relation is a function.

Domain: \([-2,3]\) or \(-2 \leq x < 3\)  
Range: \((-5, 2]\) or \(-5 < y \leq 2\)

This graph would pass the vertical line test, therefore it IS a function.

Scan this QR code to go to a video tutorial on functions, domain and range.

Slope

The slope of a line is determined by the vertical change divided by the horizontal change (or rise over run). Slope can be positive, negative, zero, or undefined.
You can determine slope by counting rise over run, or using the formula: \( m = \frac{y_2 - y_1}{x_2 - x_1} \)

**Example 3:** Find the slope of the line that passes through (-3, 9) and (2, 4).

\[
m = \frac{4 - 9}{2 - (-3)} = \frac{-5}{5} = -1
\]

**Example 4:** Find the slope of the line that passes through (-4, 3) and (-4, 0).

\[
m = \frac{0 - 3}{-4 - (-4)} = \frac{-3}{0} = undefined
\]

The intercept of a line is where it crosses the axis. The x-intercept is where a line crosses the x-axis, and the y-intercept is where a line crosses the y-axis.

A special form of a linear equation is called slope-intercept form: \( y = mx + b \). Where \( m \) is the slope of the line and \( b \) is the y-intercept (0, b).

It is often easier to graph an equation when it is written in slope-intercept form. You can transform an equation into slope-intercept form by solving for \( y \).

**Example 5:** Put in slope intercept form, then state the slope and the y-intercept.

\[
12x - 3y = 9
\]

\[
-12x
\]

\[
-3y = -12x + 9
\]

\[
\div (-3)
\]

\[
y = 4x - 3
\]

The slope is 4, and the y-intercept is (0, -3).

To graph the equation above, put a point on the y-axis at -3, and then count the slope by going up 4 and to the right 1. Put a second point there and draw a line through the two points.
Another special form of a linear equation is point-slope form: \( y - y_1 = m(x - x_1) \)
Where \( m \) is the slope and \((x_1, y_1)\) is a point on the line.

If you are given two points and asked to write the equation of the line, first use the two points to solve for the slope, then write the equation in point-slope form using either point.

**Example 6:** A line passes through \((3, 0)\) and \((-5, 2)\). What is an equation of the line?

\[
m = \frac{2 - 0}{-5 - 3} = \frac{2}{-8} = -\frac{1}{4}
\]

\[
y - 0 = -\frac{1}{4}(x - 3)
\]

Sometimes you will be asked to find a line that best fits some given data. This can be done in the calculator.

**Example 7:** The cost of a gallon of gas for the past 6 years is given. Write an equation for the line of best fit, then use this equation to predict gas prices in 2017.

<table>
<thead>
<tr>
<th>Year</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost for one gallon</td>
<td>2.59</td>
<td>3.16</td>
<td>3.29</td>
<td>3.25</td>
<td>3.51</td>
<td>3.56</td>
</tr>
</tbody>
</table>

Start by entering this data into the list in your calculator. 2006 can be year 1, 2007 can be year 2, etc.

Then hit the stat button again, scroll over to calc, and select number 4 (LinReg) 

Press enter twice and your results window will show.

The line of best fit is \( y = .167x + 2.64 \)

To answer the second part of the question, we first need to determine what number the year 2017 would be associated with. Since 2011 was year 6, 2017 would be year 12.
To predict the gas price in 2017, we will plug 12 in for \( x \) in our line of best fit.

\[
y = .167 \times 12 + 2.64 = 4.64
\]
1. Find the slope of the line that passes through (0, -4) and (-2, 5).

2. Write the equation of a line that has a slope of 5 and passes through (2, -3).

3. Write the equation of the line that passes through (7, -1) and (2, 4) in slope-intercept form, then graph the line.

4. The number of hours 6 students spent studying and their quiz grade are recorded below. Write an equation for the line of best fit, then use this equation to predict your score if you studied for 0.75 hours.

<table>
<thead>
<tr>
<th>Hours</th>
<th>2</th>
<th>0.5</th>
<th>3</th>
<th>1</th>
<th>0.5</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiz Grade</td>
<td>88</td>
<td>84</td>
<td>96</td>
<td>90</td>
<td>77</td>
<td>91</td>
</tr>
</tbody>
</table>

5. Using the equation from problem 4, how long would a student need to study to get a 100 on the quiz?

### Absolute Value Functions

**All.4** The student will solve, algebraically and graphically, a) absolute value equations and inequalities;

**All.6** The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

**All.7** The student will investigate and analyze functions algebraically and graphically.

The absolute value of a number, $x$, is its distance from zero on a number line, and is written $|x|$.

If you are told $|x| = 4$, then $x$ could equal 4 or -4 since $|4|$ and $|-4|$ both equal 4. Notice that both 4 and -4 are 4 units from zero on the number line below.
Example 1: Solve for x: \(-2 |x - 4| + 1 = -11\)

\[
\begin{align*}
-2 |x - 4| + 1 &= -11 \\
-2 |x - 4| &= -12 \\
|x - 4| &= 6
\end{align*}
\]

\[
\begin{align*}
x - 4 &= 6 & \text{or} & & x - 4 &= -6 \\
+4 & & +4 & & +4 & & +4
\end{align*}
\]

\[
x = 10 & \quad \text{or} & & x = -2
\]

Don’t forget to check your work!

\[
\begin{align*}
-2 |10 - 4| + 1 &= -11 & & -2 |-2 - 4| + 1 &= -11 \\
-2 |6| + 1 &= -11 & & -2 |-6| + 1 &= -11 \\
-2 (6) + 1 &= -11 & & -2 (6) + 1 &= -11 \\
-12 + 1 &= -11 & \checkmark & & -12 + 1 &= -11 & \checkmark
\end{align*}
\]

Example 2: Solve for x: \(|x - 3| = 2x - 10\)

\[
\begin{align*}
x - 3 &= 2x - 10 & \quad \text{or} & & x - 3 &= -(2x - 10) \\
-2x & & +2x & & +2x
\end{align*}
\]

\[
\begin{align*}
-3 &= x - 10 & & 3x - 3 &= 10 \\
+10 & & +10 & & +3 & & +3
\end{align*}
\]

\[
x = 7 & \quad \text{or} & & x = \frac{13}{3}
\]

Don’t forget to check your work!

\[
\begin{align*}
|7 - 3| &= 2(7) - 10 & & \left| \frac{13}{3} - 3 \right| &= 2 \left( \frac{13}{3} \right) - 10 \\
|4| &= 14 - 10 & & \left| \frac{4}{3} \right| &= \frac{26}{3} - 10 \\
& & & \frac{4}{3} \neq \frac{-4}{3}
\end{align*}
\]

7 is the only solution that works, therefore \(\frac{13}{3}\) is an extraneous solution.

Absolute Value Inequalities

| \(|x| < a\) | \(-a < x < a\) |
| \(|x| \leq a\) | \(-a \leq x \leq a\) |
| \(|x| > a\) | \(x < -a \quad \text{or} \quad x > a\) |
| \(|x| \geq a\) | \(x \leq -a \quad \text{or} \quad x \geq a\) |
Example 3: Solve and graph the inequality. Write your answer in interval and set notation.

\[ 3|x - 6| > 9 \]
\[ \div 3 \div 3 \]
\[ |x - 6| > 3 \]

\[ x - 6 < -3 \]
\[ +6 +6 +6 +6 \]
\[ x < 3 \]
\[ (\infty, 3) \cup (9, \infty) \]

\[ x - 6 > 3 \]
\[ +6 +6 +6 +6 \]
\[ x > 9 \]
\[ or \{ x | x < 3 or x > 9 \} \]

Transformations of Functions

<table>
<thead>
<tr>
<th>Vertical Translations</th>
<th>Horizontal Translations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translation up ( k ) units</td>
<td>Translation to the right ( h ) units</td>
</tr>
<tr>
<td>( y = f(x) + k )</td>
<td>( y = f(x - h) )</td>
</tr>
<tr>
<td>Translation down ( k ) units</td>
<td>Translation to the left ( h ) units</td>
</tr>
<tr>
<td>( y = f(x) - k )</td>
<td>( y = f(x + h) )</td>
</tr>
</tbody>
</table>

Vertical Stretches and Compression

<table>
<thead>
<tr>
<th>Reflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical stretch, ( a &gt; 1 )</td>
</tr>
<tr>
<td>( y = af(x) )</td>
</tr>
<tr>
<td>Vertical compression ( 0 &lt; a &lt; 1 )</td>
</tr>
<tr>
<td>( y = af(x) )</td>
</tr>
<tr>
<td>Across the ( x )-axis</td>
</tr>
<tr>
<td>( y = -f(x) )</td>
</tr>
<tr>
<td>Across the ( y )-axis</td>
</tr>
<tr>
<td>( y = f(-x) )</td>
</tr>
</tbody>
</table>

Example 4: Describe the transformation from \( f(x) = x \) to \( g(x) = -x + 1 \)

\( g(x) \) is reflected across the \( x \)-axis, and shifted up one unit.

Example 5: Graph \( y = |x + 2| + 3 \)

This graph will be translated left 2 units and up 3 units from the parent graph \( y = |x| \)
**TRY IT: Absolute Value Functions**

1. Solve for x. Check for extraneous solutions! \(|2x - 3| = x - 5\)
2. Solve and graph. Write your solution in interval and set notation.
   \(|6x - 11| \leq 7\)
3. Solve and graph. Write your solution in interval and set notation.
   \(-2|6x - 1| + 5 < 3\)
4. Describe the transformation from \(f(x)\) to \(g(x)\).
   \(f(x) = x^2 \quad g(x) = \frac{1}{2}x^2 + 5\)
4. Describe the transformation from \(f(x)\) to \(g(x)\).
   \(f(x) = |x| \quad g(x) = -2|x - 5|\)

**Quadratic Functions**

**All/T.7** The student will investigate and analyze functions algebraically and graphically.

Key concepts include
- a) domain and range, including limited and discontinuous domains and ranges;
- b) zeros;
- c) \(x\)- and \(y\)-intercepts;
- d) intervals in which a function is increasing or decreasing;

**All/T.4** The student will solve, algebraically and graphically,
- b) quadratic equations over the set of complex numbers;

Standard form for a quadratic function is: \(f(x) = ax^2 + bx + c, \quad a \neq 0\)
If \(a > 0\) then the parabola opens upward. If \(a < 0\) then the parabola opens downward.

The axis of symmetry is the line \(x = \frac{-b}{2a}\).

The \(x\)-coordinate of the vertex is \(\frac{-b}{2a}\). The \(y\)-coordinate of the vertex is found by plugging that \(x\) value into the equation and solving for \(f(x)\).
The \(y\)-intercept is \((0, c)\).

To graph a quadratic:
1. Identify \(a\), \(b\), and \(c\).
2. Find the axis of symmetry \(x = \frac{-b}{2a}\), and lightly sketch.
3. Find the vertex. The \(x\)-coordinate is \(\frac{-b}{2a}\). Use this to find the \(y\)-coordinate.
4. Plot the \(y\)-intercept \((c)\), and its reflection across the axis of symmetry.
5. Draw a smooth curve through your points.
The vertex of a parabola is its turning point. In the parabola pictured below the function is decreasing from $(-\infty, 2)$, and the function is increasing from $(2, \infty)$. Notice that we use parenthesis to indicate that the parabola is not increasing or decreasing at 2.

Example 1: Graph $y = 2x^2 - 4x + 3$

Step 1: Identify $a$, $b$, and $c$. $a = 2$, $b = -4$, and $c = 3$

Step 2: Find and sketch the axis of symmetry.

$$x = \frac{-b}{2a} \quad x = \frac{-(-4)}{2(2)} \quad x = \frac{4}{4} \quad x = 1$$

Step 3: Find the vertex.

The $x$-coordinate is 1. Plug this in to find $y$.

$$y = 2(1)^2 - 4(1) + 3 \quad y = 2 - 4 + 3 \quad y = 1$$

The vertex is $(1, 1)$.

Step 4: Plot the $y$-intercept and its reflection.

Because $c = 3$, the $y$-intercept is $(0, 3)$. Reflecting this point across $x = 1$ gives the point $(2, 3)$.

Step 5: Draw a smooth curve.

Factoring Quadratics

To factor a trinomial of the form $x^2 + bx + c = 0$, first find two integers whose sum is equal to $b$, and whose product is equal to $a \cdot c$.

You can start by listing all of the factors of $a \cdot c$, and then see which two factors sum up to the coefficient of $b$.

Once you have determined which factors to use, you can put all of your terms “in a box” and factor the rows and columns. This is one method that can be used to factor.
Example 2: Factor $x^2 + 6x + 8$

So, we are looking for factors of 8 that add up to 6!

<table>
<thead>
<tr>
<th>Factors of 8</th>
<th>Sum of factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 8</td>
<td>9</td>
</tr>
<tr>
<td>2, 4</td>
<td>6 ✔</td>
</tr>
</tbody>
</table>

Put terms “in a box”

Find the greatest common factor in each row and each column. These will give you your two binomials!

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$x^2$</td>
</tr>
<tr>
<td>4</td>
<td>$4x$</td>
</tr>
</tbody>
</table>

$(x + 4)(x + 2)$

Check your answer by FOIL-ing!

When factoring, anytime the b term is negative and the c term is positive, your answer will have two minus signs!

Example 3: Factor $8x^2 - 21x + 10$

So, we are looking for factors of 80 that add up to $-21$!

<table>
<thead>
<tr>
<th>Factors of 80</th>
<th>Sum of factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4, -20</td>
<td>-24</td>
</tr>
<tr>
<td>-5, -16</td>
<td>-21  ✔</td>
</tr>
</tbody>
</table>

Find the greatest common factor in each row and each column. These will give you your two binomials!

<table>
<thead>
<tr>
<th>$8x$</th>
<th>$-5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$8x^2$</td>
</tr>
<tr>
<td>$-2$</td>
<td>$-16x$</td>
</tr>
</tbody>
</table>

$(8x - 5)(x - 2)$

Check your answer by FOIL-ing!
Special Cases

A perfect square trinomial can be factored to two binomials that are the same, so you can write it as the binomials squared.

\[ a^2 + 2ab + b^2 = (a + b)^2 \quad a^2 - 2ab + b^2 = (a - b)^2 \]

**Example 4:** Factor \( 4x^2 - 24x + 36 \)

If your first and last terms are perfect squares, you can check for a perfect square trinomial. Take the square root of the first and last number and see if the product of those is equal to \( \frac{1}{2} \) of the middle number.

\[ \sqrt{4} = 2 \quad \text{and} \quad \sqrt{36} = 6 \quad 6 \cdot 2 = 12, \text{which is } \frac{1}{2} \text{ of 24} \]

Now that we know this case works, you can write the binomial factor squared

\[ (2x - 6)^2 \]

Remember to check your answer by FOIL-ing the binomials back out!

Another special case is if the quadratic is represented as the difference of two perfect squares (i.e. \( 4x^2 - 16 \)). If both the first and last term are perfect squares and the two terms are being subtracted, then their factorization can be written as \( (a + b)(a - b) \). As an example, \( 4x^2 - 16 = (2x + 4)(2x - 4) \). Remember that you can check your work by FOIL-ing.

**Example 5:** Factor completely \( 3x^2 - 27 \)

To begin, you should factor out a GCF. In this case it would be 3.

\[ 3(x^2 - 9) \quad \text{Now you are left with a difference of squares!} \]

\[ 3(x + 3)(x - 3) \]
To solve a quadratic equation (i.e. find its solutions, roots, or zeros), set one side equal to zero (put the quadratic in standard form), then factor. Set each factor equal to zero to find the values for x that are the solutions to the quadratic.

**Example 6:** Find the zeros of \( x^2 - 18 = 7x \)

Start by getting one side equal to zero and write in standard form.

\[
\begin{align*}
  x^2 - 18 &= 7x \\
  -7x &\quad -7x \\
  x^2 - 7x - 18 &= 0
\end{align*}
\]

Now factor the trinomial.

We are looking for factors of \(-18\) that add up to \(-7\). \(-9\) and \(2\) work!

\[
\begin{array}{c|cc}
  x & x^2 & -9x \\
  \hline
  2 & 2x & -18 \\
  -9 & \hline
\end{array}
\]

\[(x + 2)(x - 9) = 0 \quad \text{Set both factors equal to zero!} \]

\[
x + 2 = 0 \quad \text{and} \quad x - 9 = 0 \\
x = -2 \quad \text{and} \quad 9
\]

You can check your answer in your calculator by graphing the quadratic. The solutions are the x-intercepts, so this graph should cross the x-axis at -2 and 9.
Sometimes you will need to solve a quadratic that cannot be factored. In that case you can use the quadratic formula: 

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

You just substitute the values for a, b, and c into the quadratic formula and simplify.

**Example 7:** Solve \(5x^2 - 2x - 9 = 0\)

\[a = 5 \quad b = -2 \quad c = -9\]  Plug these values into the quadratic formula

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad x = \frac{2\pm\sqrt{(-2)^2 - 4(5)(-9)}}{2(5)} \quad x = \frac{2\pm\sqrt{184}}{10} \quad x = \frac{2\pm\sqrt{184}}{10}\]

Your two solutions are \(x = \frac{2+\sqrt{184}}{10}\) and \(x = \frac{2-\sqrt{184}}{10}\).

In Example 7, the quadratic had two solutions, but a quadratic can also have only one solution or no solutions at all.

You can determine how many solutions a quadratic equation will have by calculating the discriminant \(b^2 - 4ac\). This is the part that is located under the square root in the quadratic formula. If the discriminant is positive, the quadratic will have two real solutions. If the discriminant is zero, the quadratic will have one real solution. If the discriminant is negative, the quadratic will have no real solutions.

Scan this QR code to go to a video tutorial on finding the solutions of a quadratic.
Example 8: Determine the number of solutions of \( 7x^2 - x = 12 \)

Start by getting one side equal to zero and write in standard form.

\[
7x^2 - x - 12 = 0
\]

Now, find the discriminant.

\[
a = 7 \quad b = -1 \quad c = -12
\]

\[
b^2 - 4ac = (-1)^2 - 4(7)(-12) = 1 + 336 = 337
\]

Because the discriminant is positive, this quadratic has two solutions.

**TRY IT:** Quadratic Functions

1. Graph \( y = -\frac{1}{2}x^2 + 4x + 2 \)
2. Factor \( x^2 - 11x + 18 \)
3. Factor \( 3x^2 - 5x - 12 \)
4. Factor \( 5x^4 - 125 \)
5. Find the roots \( 9x^2 + 48x + 64 = 0 \)
6. Determine the number of solutions \( 4x^2 - 3x + 10 = 0 \)
7. Find the zeros \( x^2 - 5x = 14 \)
8. Find the solutions \( -2x^2 + 7 = -4x \)

Complex Numbers

**All/T.3** The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of \( i \), and identify field properties that are valid for the complex numbers.

**All/T.4** The student will solve, algebraically and graphically,

a) absolute value equations and inequalities;

b) quadratic equations over the set of complex numbers;
The complex numbers are made up of the real numbers and the imaginary numbers.

**Complex Numbers** $(a + bi)$

<table>
<thead>
<tr>
<th>Real Numbers $(a + 0i)$</th>
<th>Imaginary Numbers $(a + bi, b ≠ 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pure Imaginary Numbers $(0 + bi, b ≠ 0)$</td>
</tr>
</tbody>
</table>

The imaginary number, $i$, is the same as $\sqrt{-1}$  
$i = \sqrt{-1}$  
therefore $i^2 = -1$

**Example 1:** Simplify $\sqrt{-8}$

We can start by pulling $i$ out to factor the negative out from under the radical.

$$i \cdot \sqrt{8}$$

Now simplify the radical by pulling out any perfect square factors (i.e. 4).

$$i \cdot \sqrt{4 \cdot 2}$$

$$2i \sqrt{2}$$

You can also perform operations on complex numbers. Remember that if you get an $i^2$ term that it can be replaced with $-1$.

**Example 2:** Simplify $(3 - 3i) - (-4 + 5i)$  
Don’t forget to distribute the negative!!

$$3 - 3i + 4 - 5i$$

And then combine like terms!

$$7 - 8i$$

**Example 3:** Simplify $(3 - 3i)(-4 + 5i)$  
You can FOIL this!

$$-12 + 15i + 12i - 15i^2$$

Combine like terms

$$-12 + 27i - 15i^2$$

Replace $i^2$ with $-1$

$$-12 + 27i - 15(-1)$$

Simplify

$$-12 + 27i + 15$$

$$3 + 27i$$
Number pairs that are of the form \( a + bi \) and \( a - bi \) are called **complex conjugates**. When you multiply complex conjugates, the product is a real number. This can be used to simplify quotients of complex numbers.

**Example 4:** Simplify \((9 - 5i)(9 + 5i)\)

\[
81 + 45i - 45i - 25i^2 \\
81 - 25i^2 \\
81 - 25(-1) \\
81 + 25 \\
106
\]

**Example 5:** Find the quotient by using the complex conjugate.

\[
\frac{8-16i}{4i} \\
\frac{8-16i}{4i} \cdot \frac{-4i}{-4i} \\
\frac{-32i+64i^2}{-16i^2} \\
\frac{-32i+64(-1)}{-16(-1)} \\
\frac{-64-32i}{16} = -4 - 2i \\
\]

**TRY IT:** **Complex Numbers**

Simplify

1. \(\sqrt{-32x}\)
2. \((6 - 4i) + (2 - 3i) - (6 - 6i)\)
3. \(-5i(4 + 3i)\)
4. \((2 + i)(3 - 2i)\)
5. \(\frac{6-10i}{-2i}\)
6. \(\frac{4+7i}{1-3i}\)
Answers to the \textbf{TRY IT} problems:

\textbf{Linear Equations and Inequalities}
1. \( y = -28 \)
2. \( n = -\frac{1}{2} \)
3. \( k = 4 \)
4. \( g = -1 \)
5. \( m = -\frac{5}{3} \)
6. \( x = 2 \)
7. \( x > 5 \)
8. \( w \leq 8 \)
9. \( g > 3 \)
10. \((-9,10]\)
11. \([20,45]\)
12. \textit{sales > $1900}

\textbf{Absolute Value Functions}
1. No Solution (Both are extraneous)
2. \( \{x \mid \frac{2}{3} \leq x \leq 3\} \text{ or } [\frac{2}{3}, 3] \)

\begin{figure}[h]
\begin{center}
\includegraphics[width=0.5\textwidth]{absolute_value_graph}
\end{center}
\end{figure}

3. \( \{x \mid x > \frac{1}{3} \text{ or } x < 0\} \text{ or } (-\infty, 0) \cup (\frac{1}{3}, \infty) \)

\begin{figure}[h]
\begin{center}
\includegraphics[width=0.5\textwidth]{absolute_value_graph_2}
\end{center}
\end{figure}

4. Vertically compressed by \( \frac{1}{2} \), and shifted up 5 units.
5. Reflected across the x-axis, vertically stretched by 2, and shifted to the right 5 units.

\textbf{Quadratic Functions}
1. \( (x - 2)(x - 9) \)
2. \( (3x + 4)(x - 3) \)
3. \( 5(x^2 + 5)(x^2 - 5) \)
4. \( x = -\frac{8}{3} \)
5. \( x = -2, 7 \)
6. \( x \approx -1.121, 3.121 \)

\textbf{Complex Numbers}
1. \( 4i\sqrt{2}x \)
2. \( 2 - i \)
3. \( 15 - 20i \)
4. \( 8 - i \)
5. \( 5 + 3i \)
6. \( \frac{-17 + 19i}{10} \)