Geometry Content Review Notes are designed by the High School Mathematics Steering Committee as a resource for students and parents. They have been revised this year as part of an internship process. Each nine weeks’ Standards of Learning (SOLs) have been identified and a detailed explanation of the specific SOL is provided. Specific notes have also been included in this document to assist students in understanding the concepts. Sample problems allow the students to see step-by-step models for solving various types of problems. A “TRY IT” section has also been developed to provide students with the opportunity to solve similar problems and check their answers.

The document is a compilation of information found in the Virginia Department of Education (VDOE) Curriculum Framework, Enhanced Scope and Sequence, and Released Test items. In addition to VDOE information, Prentice Hall Textbook Series and resources have been used. Finally, information from various websites is included. The websites are listed with the information as it appears in the document.

Supplemental online information can be accessed by scanning QR codes throughout the document. These will take students to video tutorials and online resources. In addition, a self-assessment is available at the end of the document to allow students to check their readiness for the nine-weeks test.

To access the database of online resources scan this QR code, or visit http://spsmath.weebly.com

The Geometry Blueprint Summary Table is listed below as a snapshot of the reporting categories, the number of questions per reporting category, and the corresponding SOLs.

<table>
<thead>
<tr>
<th>Reporting Categories</th>
<th>No. of Items</th>
<th>SOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lines and Angles</td>
<td>11</td>
<td>G.3, G.4, G.11</td>
</tr>
<tr>
<td>Triangles and Logic</td>
<td>12</td>
<td>G.1a,b,c,d, G.5a,b</td>
</tr>
<tr>
<td>Polygons and Circles</td>
<td>10</td>
<td>G.8a,b,c, G.9, G.10</td>
</tr>
<tr>
<td>Three-dimensional Figures</td>
<td>6</td>
<td>G.12, G.13, G.14a,b</td>
</tr>
<tr>
<td>Coordinate Relations and Transformations</td>
<td>6</td>
<td>G.2a,b,c</td>
</tr>
<tr>
<td>Total Number of Operational Items</td>
<td>45</td>
<td></td>
</tr>
<tr>
<td>Field Test Items*</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Total Number of Items</td>
<td>55</td>
<td></td>
</tr>
</tbody>
</table>

*These field test items will not be used to compute students’ scores on the tests.
Geometry Formula Sheet
2009 Mathematics Standards of Learning

Geometric Formulas

\[ A = \frac{1}{2}bh \]
\[ A = \frac{1}{2}ab \sin C \]
\[ p = 4s \]
\[ A = s^2 \]
\[ p = 2l + 2w \]
\[ A = lw \]
\[ C = 2\pi r \]
\[ A = \pi r^2 \]

\[ A = bh \]
\[ A = \frac{1}{2}h(b_1 + b_2) \]
\[ V = Bh \]
\[ L.A. = hp \]
\[ S.A. = hp + 2B \]

\[ V = \pi r^2h \]
\[ L.A. = 2\pi rh \]
\[ S.A. = 2\pi r^2 + 2\pi rh \]

\[ V = \frac{4}{3}\pi r^3 \]

\[ V = \frac{1}{3}\pi r^2h \]
\[ L.A. = \pi rl \]
\[ S.A. = \pi r^2 + \pi rl \]

\[ V = \frac{1}{3}Bh \]

\[ L.A. = \frac{1}{2}lp \]
\[ S.A. = \frac{1}{2}lp + B \]

Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Symbol</th>
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<tbody>
<tr>
<td>Area</td>
<td>( A )</td>
</tr>
<tr>
<td>Area of Base</td>
<td>( B )</td>
</tr>
<tr>
<td>Circumference</td>
<td>( C )</td>
</tr>
<tr>
<td>Lateral Area</td>
<td>( L.A. )</td>
</tr>
<tr>
<td>Perimeter</td>
<td>( p )</td>
</tr>
<tr>
<td>Surface Area</td>
<td>( S.A. )</td>
</tr>
<tr>
<td>Volume</td>
<td>( V )</td>
</tr>
</tbody>
</table>

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Geometry Formula Sheet
2009 Mathematics Standards of Learning

Geometric Formulas

\[ a^2 + b^2 = c^2 \]
\[ \sin \theta = \frac{y}{z} \]
\[ \cos \theta = \frac{x}{z} \]
\[ \tan \theta = \frac{y}{x} \]
\[ (x-h)^2 + (y-k)^2 = r^2 \]
\[ \pi \approx 3.14 \]
\[ \pi \approx \frac{22}{7} \]

Quadratic Formula:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } ax^2 + bx + c = 0 \text{ and } a \neq 0 \]

Geometric Symbols

<table>
<thead>
<tr>
<th>Example</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m \angle A )</td>
<td>measure of angle ( A )</td>
</tr>
<tr>
<td>( AB )</td>
<td>length of line segment ( AB )</td>
</tr>
<tr>
<td>( \overrightarrow{AB} )</td>
<td>ray ( AB )</td>
</tr>
<tr>
<td>( \perp )</td>
<td>right angle</td>
</tr>
<tr>
<td>( AB \parallel CD )</td>
<td>Line ( AB ) is parallel to line ( CD ).</td>
</tr>
<tr>
<td>( AB \perp CD )</td>
<td>Line segment ( AB ) is perpendicular to line segment ( CD ).</td>
</tr>
<tr>
<td>( \angle A \cong \angle B )</td>
<td>Angle ( A ) is congruent to angle ( B ).</td>
</tr>
<tr>
<td>( \triangle ABC \sim \triangle DEF )</td>
<td>Triangle ( ABC ) is similar to triangle ( DEF ).</td>
</tr>
<tr>
<td></td>
<td>Similarly marked segments are congruent.</td>
</tr>
<tr>
<td></td>
<td>Similarly marked angles are congruent.</td>
</tr>
</tbody>
</table>
Algebra Review

Solving Equations

You will solve an **equation** to find all of the possible values for the variable. In order to solve an equation, you will need to **isolate the variable** by performing **inverse operations** (or ‘undoing’ what is done to the variable).

Any operation that you perform on one side of the equal sign **MUST** be performed on the other side as well. Drawing an arrow down from the equal sign may help remind you to do this.

**Example 1:**

\[ m - 9 = -3 \]

\[ +9 \quad +9 \]

\[ m = 6 \]

Check your work by plugging your answer back in to the original problem.

\[ 6 - 9 = -3 \quad \checkmark \]

**Example 2:**

\[ \frac{x+4}{5} = -12 \]

\[ \cdot 5 \quad \cdot 5 \]

\[ x + 4 = -60 \]

\[ -4 \quad -4 \]

\[ x = -64 \]

Check your work by plugging your answer back in to the original problem.

\[ \frac{-64+4}{5} = \frac{-60}{5} = -12 \quad \checkmark \]
You may have to distribute a constant and combine like terms before solving an equation.

Example 3:
\[-4(g - 7) + 2g = -10\]
\[-4g + 28 + 2g = -10\]
\[-2g + 28 = -10\]
\[-28 = -28\]
\[-2g = -38\]
\[\div (-2) \quad \div (-2)\]

\[g = 19\]

Check your work by plugging your answer back in to the original problem.

\[-4 \left(19 - 7\right) + 2(19) = -10\]
\[-4(12) + 2(19) = -10\]
\[-48 + 38 = -10 \checkmark\]

If there are variables on both sides of the equation, you will need to move them all to the same side in the same way that you move numbers.

Example 4:
\[3p - 5 = 7(p - 3)\]
\[3p - 5 = 7p - 21\]
\[-3p \quad - 3p\]
\[-5 = 4p - 21\]
\[+21 \quad + 21\]
\[16 = 4p\]
\[\div 4 \quad \div 4\]

\[p = 4\]

Check your work by plugging your answer back in to the original problem.

\[3(4) - 5 = 7(4) - 21\]
\[12 - 5 = 28 - 21\]
\[7 = 7 \checkmark\]
Example 5: \[
\frac{x+10}{5x} = \frac{-1}{5}
\]
You can begin this problem by cross multiplying!

\[
5(x + 10) = -1(5x)
\]
\[
5x + 50 = -5x
\]
\[
+5x \quad +5x
\]
\[
10x + 50 = 0
\]
\[
-50 \quad -50
\]
\[
10x = -50
\]
\[
\div 10 \quad \div 10
\]
\[
x = -5
\]

Check your work by plugging your answer back in to the original problem.

\[
\frac{-5+10}{5(-5)} = \frac{-1}{5}
\]
\[
\frac{5}{-25} = \frac{-1}{5}
\]
\[
-\frac{1}{5} = -\frac{1}{5} \checkmark
\]

**TRY IT: Solving Equations**

Solve each equation

1. \(k + 11 = -8\)
2. \(9 - 3x = 54\)
3. \(-17 = \frac{y-6}{2}\)
4. \(5(2n + 6) + 8 = 33\)
5. \(3 - (4k + 2) = -15\)
6. \(5g + 4 = -9g - 10\)
7. \(-2(-4m - 1) + 3m = 4m - 8 + m\)
8. \(\frac{x-4}{2} = \frac{-2(3x-3)}{6}\)
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point</td>
<td>A point is used to signify location only. A point has no length, width or depth. A point is represented by a dot and labeled with a capital letter.</td>
<td><img src="https://via.placeholder.com/150" alt="Points" /></td>
</tr>
<tr>
<td>Line</td>
<td>A line has only one dimension: length. It continues indefinitely in both directions and is made up of infinitely many points. It can be labeled with a small script letter or by any two points on the line. (i.e. line $v$, $\overrightarrow{PQ}$, $\overrightarrow{QP}$)</td>
<td><img src="https://via.placeholder.com/150" alt="Lines" /></td>
</tr>
<tr>
<td>Collinear Points</td>
<td>Points that lie on the same line</td>
<td><img src="https://via.placeholder.com/150" alt="Collinear Points" /></td>
</tr>
<tr>
<td>Non-collinear Points</td>
<td>Points that do NOT lie on the same line</td>
<td><img src="https://via.placeholder.com/150" alt="Non-collinear Points" /></td>
</tr>
<tr>
<td>Plane</td>
<td>Thought of as a two dimensional (or flat) object that extends indefinitely in all directions. It can be labeled by a capital script letter or by three non-collinear points. (i.e. $\mathcal{R}$, ABC, BCA, ACB, CBA)</td>
<td><img src="https://via.placeholder.com/150" alt="Planes" /></td>
</tr>
<tr>
<td>Co-Planar</td>
<td>Points that lie on the same plane</td>
<td><img src="https://via.placeholder.com/150" alt="Co-Planar" /></td>
</tr>
<tr>
<td>Non Co-planar</td>
<td>Points that do NOT lie on the same plane.</td>
<td><img src="https://via.placeholder.com/150" alt="Non Co-planar" /></td>
</tr>
<tr>
<td>Line Segment</td>
<td>A portion of a line that is defined by two points. It has a finite length and no width. It is labeled by the two endpoints. (i.e. $\overline{MN}$, $\overline{NM}$)</td>
<td><img src="https://via.placeholder.com/150" alt="Line Segment" /></td>
</tr>
</tbody>
</table>
Segments, Midpoint and Distance

**G.3** The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include a) investigating and using formulas for finding distance, midpoint, and slope;

The length of a line segment is determined by finding the distance between the two endpoints. Because a distance is always positive, to find the length you will take the absolute value of the difference between the two endpoints. \( MN = |x_1 - x_2| \).

**Example 1:** Find the length of segment AB.

\[
AB = |x_1 - x_2| \\
AB = |-2 - 3| \\
AB = |-5| \\
AB = 5
\]

**Example 2:** Find the distance between the table and the tree.

\[
74 - 26 = 48 \text{ feet}
\]

The distance between two ordered pairs can be determined using the distance formula.

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

You might find it helpful to label your points \((x_1, y_1)\) & \((x_2, y_2)\) before starting.

Scan this QR code to go to a video tutorial on using the distance formula.
Example 3: Find the distance between X and Y if: X (3, -1) and Y (0, -4)

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ d = \sqrt{(0 - 3)^2 + (-4 - (-1))^2} \]
\[ d = \sqrt{(-3)^2 + (-3)^2} \]
\[ d = \sqrt{9 + 9} \]
\[ d = \sqrt{18} \approx 4.24 \]

***Honors Geometry***

Simplifying Radicals

In the last example, our answer was \( \sqrt{18} \). This radical can be simplified.

To simplify a radical, you will pull out any perfect square factors (i.e. 4, 9, 16, 25, etc.)

\[ \sqrt{18} = \sqrt{9 \cdot 2} \]

The square root of 9 is equal to 3, so you can pull the square root of 9 from underneath the radical sign to find the simplified answer \( 3\sqrt{2} \), which means 3 times the square root of 2. You can check this simplification in your calculator by verifying that \( \sqrt{18} = 3\sqrt{2} \).

Another way to simplify radicals, if you don’t know the factors of a number, is to create a factor tree and break the number down to its prime factors. When you have broken the number down to all of its prime factors you can pull out pairs of factors, which will multiply together to make perfect squares.

Example: Simplify \( \sqrt{128} \)

\[ \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} \]

\[ 2 \cdot 2 \cdot 2 \sqrt{2} = 8\sqrt{2} \]

Scan this QR code to go to a video tutorial on simplifying radicals.
The midpoint of a line segment can be found using the formula \( \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \).

Example 4: Find the midpoint between (4, -6) and (10, 2).

\[
\begin{align*}
(x_1, y_1) & \quad (x_2, y_2) \\
(4, -6) & \quad (10, 2) \\
\left( \frac{4 + 10}{2}, \frac{-6 + 2}{2} \right) \\
\left( \frac{14}{2}, \frac{-4}{2} \right) & = (7, -2)
\end{align*}
\]

Example 5: Given that M is the midpoint of \( \overline{PQ} \), find \( x \) and determine the length of \( \overline{PQ} \).

If M is the midpoint of \( \overline{PQ} \), then \( PM = MQ \) so

\[
\begin{align*}
5x - 4 & = 2x + 8 \\
-2x & = -2x \\
3x - 4 & = 8 \\
+4 & = +4 \\
3x & = 12 \\
\div 3 & \div 3 \\
x & = 4
\end{align*}
\]

If \( x = 4 \), then \( PM = 5(4) - 4 = 20 - 4 = 16 \) and \( MQ = 2(4) + 8 = 8 + 8 = 16 \). Therefore, \( PQ = PM + MQ \) or \( PQ = 16 + 16 = 32 \).

Scan this QR code to go to a video tutorial on finding and using the midpoint.
**TRY IT: Segments, Midpoint and Distance**

1. Find $x$, $PQ$, and $QR$. 

2. Find the distance between $(9, 2)$ and $(-1, 4)$.

3. Find $CD$ if $M$ is the midpoint.

4. If the midpoint of line segment $XY$ is $(0, -3)$ and $X$ is at $(4, -2)$, find the coordinates of $Y$.

**Writing Equations of Circles**

G.11 The student will use angles, arcs, chords, tangents, and secants to 
   a) investigate, verify, and apply properties of circles;
   b) solve real-world problems involving properties of circles

Standard equation of a circle: $x^2 + y^2 = r^2$ where $r$ is the radius of the circle, and the center of the circle is located at the origin $(0, 0)$.

When the center is not at the origin, you have to apply a shift to the $x$ and $y$ coordinates. 

$$(x - h)^2 + (y - k)^2 = r^2$$

$h$ is the shift in the $x$-coordinate, and $k$ is the shift in the $y$-coordinate. Now, the center is located at $(h, k)$ and $r$ is still the radius.

**Example 1:** Identify the center and radius of $x^2 + (y + 3)^2 = 36$

The center is at $(0, -3)$, and the radius is 6
Example 2: Write the equation of a circle with center \((1, -5), \ r = \sqrt{5}\)

\[(x - 1)^2 + (y + 5)^2 = 5\]

Example 3: Write the equation of the pictured circle.

\[(h, k) = (-3, 4) \quad r = 4\]

\[(x + 3)^2 + (y - 4)^2 = 16\]

TRY IT: Writing Equations of Circles

Identify the center and radius of each circle.
1. \((x - 1)^2 + (y + 1)^2 = 49\)
2. \((x + 4)^2 + y^2 = 8\)

Write the equation of each circle.
3. center \((3, -2), r = 11\)
4. center \((-1, -1), r = 1\)
5. 6. Graph the circle given the information.
   Center \((0, 4), r = 3\)
Angles

G.4 The student will construct and justify the constructions of

c) a perpendicular to a given line from a point not on the line;
d) a perpendicular to a given line at a given point on the line;
e) the bisector of a given angle,

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
<th>Example</th>
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</thead>
<tbody>
<tr>
<td>Ray</td>
<td>Has one endpoint and extends indefinitely in the other direction. Named by endpoint and another point on the ray (endpoint must be first!) (i.e. $\overline{XY}$)</td>
<td></td>
</tr>
<tr>
<td>Opposite Rays</td>
<td>Two rays on the same line that point in opposite directions. (i.e. $\overline{BA}$ and $\overline{BC}$)</td>
<td></td>
</tr>
<tr>
<td>Angle</td>
<td>Formed by two non-collinear rays. Can be named by 3 points (vertex in the middle), vertex only (if only one angle is associated with that vertex), or with a number on the interior of the angle. (i.e. $\angle DEF$, $\angle FED$, $\angle E$, $\angle A$)</td>
<td></td>
</tr>
</tbody>
</table>

Angle Classifications

<table>
<thead>
<tr>
<th>Acute</th>
<th>Right</th>
<th>Obtuse</th>
<th>Straight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measures less than 90°</td>
<td>Measures exactly 90°</td>
<td>Measures between 90° and 180°</td>
<td>Measures exactly 180°</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle Bisector</th>
<th>A ray that divides an angle into two congruent halves (i.e. $\overline{YA}$ bisects $\angle XYZ$. Therefore $\angle AYZ \cong \angle AYX$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjacent Angles</td>
<td>Two angles that lie on the same plane, have a common vertex, and a common side.</td>
</tr>
<tr>
<td><strong>Angle Type</strong></td>
<td>Description</td>
</tr>
<tr>
<td>----------------</td>
<td>-------------</td>
</tr>
<tr>
<td><strong>Vertical angles</strong></td>
<td>Non adjacent angles formed by two intersecting lines. Vertical angles are congruent. (i.e. $\angle 1$ and $\angle 3$ are vertical and $\angle 2$ and $\angle 4$ are vertical) <em>Therefore</em> $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$</td>
</tr>
<tr>
<td><strong>Linear Pair</strong></td>
<td>Adjacent angles whose common side is a line. (i.e. $\angle 1$ and $\angle 2$ form a linear pair)</td>
</tr>
<tr>
<td><strong>Complementary Angles</strong></td>
<td>Pair of angles whose sum is 90° (i.e. $\angle \text{GHK}$ and $\angle \text{KHJ}$ are complementary)</td>
</tr>
<tr>
<td><strong>Supplementary Angles</strong></td>
<td>Pair of angles whose sum is 180° (i.e. $\angle 5$ and $\angle 6$ are supplementary angles)</td>
</tr>
<tr>
<td><strong>Perpendicular Lines</strong></td>
<td>Intersecting lines that form two right angles. (i.e. $\overline{AB} \perp \overline{XY}$)</td>
</tr>
</tbody>
</table>

$m\angle 5 = 120^\circ$  $m\angle 6 = 60^\circ$
Example 1: Find the measure of two supplementary angles if

\[ m\angle 1 = 7x \quad \text{and} \quad m\angle 2 = 3x - 10 \]

Because these angles are supplementary, we know that their sum equals 180°.

\[
7x + (3x - 10) = 180 \\
10x - 10 = 180 \\
+10 \quad +10 \\
10x = 190 \\
\div 10 \quad \div 10 \\
x = 19
\]

Once we know what x is equal to, we can solve for the measure of each angle!

\[ m\angle 1 = 7x = 7(19) = 133° \]
\[ m\angle 2 = 3x - 10 = 3(19) - 10 = 57 - 10 = 47° \]

Now you can verify that these are supplementary.

\[ 133° + 47° = 180° \]

Example 2: Find the value of x that will make \( \overrightarrow{BE} \perp \overrightarrow{AD} \)

If \( \overrightarrow{BE} \perp \overrightarrow{AD} \), then \( m\angle AFB = 90° \)

\[
6x + 18 = 90 \\
-18 \quad -18 \\
6x = 72 \\
\div 6 \quad \div 6 \\
x = 12
\]

Try It: Angles

1. In the above diagram, given that \( \overrightarrow{BE} \perp \overrightarrow{AD} \) if \( \angle BFC = 51° \), what is the measure of \( \angle CFD \)?
2. Solve for \( x \): \( \angle 1 \) is complementary to \( \angle 2 \), \( m\angle 1 = 27 - 2x \) and \( m\angle 2 = 4x - 13 \)
3. Is it possible for two acute angles to be supplementary? Explain.
**Parallel Lines and Transversals**

**G.2** The student will use the relationships between angles formed by two lines cut by a transversal to
a) determine whether two lines are parallel;
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs;

**G.3** The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include
b) applying slope to verify and determine whether lines are parallel or perpendicular;

**G.4** The student will construct and justify the constructions of
a) a line segment congruent to a given line segment;
b) the perpendicular bisector of a line segment;
c) a perpendicular to a given line from a point not on the line;
d) a perpendicular to a given line at a given point on the line;
g) a line parallel to a given line through a point not on the given line.

<table>
<thead>
<tr>
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<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel</td>
<td>Coplanar lines that never intersect. These lines will have the same slope. Parallel lines are indicated with the ( \parallel ) symbol. (i.e. ( \overline{AB} \parallel \overline{CD} ))</td>
<td>A ⊃ B ⊃ C ⊃ D</td>
</tr>
<tr>
<td>Perpendicular</td>
<td>Coplanar lines that intersect at a 90° angle. These lines will have slopes that are opposite reciprocals (i.e. ( \frac{2}{3} ) and ( -\frac{3}{2} )). Perpendicular lines are indicated with the ( \perp ) symbol. (i.e. ( \ell \perp m ))</td>
<td>l ⊥ m</td>
</tr>
<tr>
<td>Skew</td>
<td>Describes lines or planes that are not coplanar, and do not intersect.</td>
<td></td>
</tr>
<tr>
<td>Transversal</td>
<td>A line that intersects two or more lines in a single plane. Two lines cut by a transversal do not always need to be parallel. Line ( \ell ) is a transversal through ( m ) and ( n ).</td>
<td></td>
</tr>
</tbody>
</table>
When a transversal intersects two lines, eight angles are formed. These angles have special properties when the two lines being intersected are parallel.

### Exterior Angles
- \( \angle a, \angle b, \angle g, \angle h \)
- \( \angle 1, \angle 2, \angle 7, \angle 8 \)

### Interior Angles
- \( \angle c, \angle d, \angle e, \angle f \)
- \( \angle 3, \angle 4, \angle 5, \angle 6 \)

### Consecutive Interior Angles
- \( \angle c \& \angle e \), \( \angle d \& \angle f \)
- \( \angle 3 \& \angle 5 \), \( \angle 4 \& \angle 6 \)

If consecutive interior angles are **supplementary**, then the two lines are parallel.

### Alternate Interior Angles
- \( \angle c \& \angle f \), \( \angle d \& \angle e \)
- \( \angle 3 \& \angle 6 \), \( \angle 4 \& \angle 5 \)

If alternate interior angles are **congruent**, then the two lines are parallel.

### Alternate Exterior Angles
- \( \angle a \& \angle h \), \( \angle b \& \angle g \)
- \( \angle 1 \& \angle 8 \), \( \angle 2 \& \angle 7 \)

If alternate exterior angles are **congruent**, then the two lines are parallel.

### Corresponding Angles
- \( \angle a \& \angle e \), \( \angle c \& \angle g \), \( \angle b \& \angle f \), \( \angle d \& \angle h \)
- \( \angle 1 \& \angle 5 \), \( \angle 3 \& \angle 7 \), \( \angle 2 \& \angle 6 \), \( \angle 4 \& \angle 8 \)

If corresponding angles are **congruent**, then the two lines are parallel.

**Example 1:** Find a value of \( x \) that will make \( a \parallel b \).

If \( a \parallel b \), then consecutive interior angles must be supplementary.

\[
2x + 1 + 135 = 180
\]
\[
2x + 136 = 180
\]
\[
-136 = -136
\]
\[
2x = 44
\]
\[
x = 22
\]

Scan this QR code to go to a video with some examples of proving lines parallel.
Slope is the change in the y coordinate, divided by the change in the x coordinate, (or rise over run). Slope is denoted by the letter m. \( m = \frac{y_2-y_1}{x_2-x_1} \)

If two lines have the same slope, they are parallel. If two lines have slopes that are negative reciprocals, they are perpendicular.

Slopes can be positive, negative, zero, or undefined.

<table>
<thead>
<tr>
<th>Positive</th>
<th>Negative</th>
<th>Zero</th>
<th>Undefined</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Positive Slope" /></td>
<td><img src="image2.png" alt="Negative Slope" /></td>
<td><img src="image3.png" alt="Zero Slope" /></td>
<td><img src="image4.png" alt="Undefined Slope" /></td>
</tr>
</tbody>
</table>

**Example 2:** Find the slope of the line that goes through (0, 4) and (2, -3).

\[
m = \frac{y_2-y_1}{x_2-x_1} = \frac{-3-4}{2-0} = \frac{-7}{2}
\]

**Example 3:** What would be the slope of a line perpendicular to the line in Example 2?

The slope of a perpendicular line will be the negative reciprocal of the slope of the original line. Therefore the slope of the perpendicular line is \( m = \frac{2}{7} \).

You can determine the slope of a line when given the equation of the line by transforming the equation for y. Once y is on the side by itself, the equation is in slope-intercept form, and the slope is the coefficient of the x term.

\[ y = mx + b \]
Example 4: What is the slope of $4x - 2y = 8$?

Transform for $y$:

$$4x - 2y = 8$$
$$-4x + 4x$$
$$-2y = -4x + 8$$
$$\div (-2) \quad \div (-2)$$
$$y = 2x - 4 \quad m = 2$$

Example 5: What would be the slope of a line perpendicular to the line in Example 4?

The slope of a perpendicular line will be the negative reciprocal of the slope of the original line. Therefore the slope of the perpendicular line is $m = -\frac{1}{2}$.

---

**TRY IT: Parallel Lines and Transversals**

1. What value of $x$ will make a $\parallel b$?

2. What is the slope of a line that would be perpendicular to the line that passes through (4, -1) and (9, 0)?

3. Given A (-1, 2) & B (3, -5), and C (7, 2) & D (0, -2). Is AB $\parallel CD$?

4. Are these two lines parallel, perpendicular, or neither?

   $$x = 3y - 8 \quad \text{and} \quad y = \frac{1}{3}x + 4$$

5. Are these two lines parallel, perpendicular, or neither?

   $$4x - 3y = 6 \quad \text{and} \quad 4y = -3x + 9$$
**Logic**

**G.1** The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include
   a) identifying the converse, inverse, and contrapositive of a conditional statement;
   b) translating a short verbal argument into symbolic form;
   c) using Venn diagrams to represent set relationships; and
   d) using deductive reasoning.

<table>
<thead>
<tr>
<th>Term</th>
<th>Symbols</th>
<th>Example</th>
</tr>
</thead>
</table>
| Conditional Statement | $P \rightarrow Q$  
If P, then Q | If the folder is blue, then it belongs to Tom.                           |
| Converse            | $Q \rightarrow P$  
If Q, then P | If the folder belongs to Tom, then it is blue.                           |
| Inverse             | $\neg P \rightarrow \neg Q$  
If not P, then not Q | If the folder is not blue, then it does not belong to Tom.               |
| Contra-positive     | $\neg Q \rightarrow \neg P$  
If not Q, then not P | If the folder does not belong to Tom, then the folder is not blue.       |
| Biconditional       | $P \leftrightarrow Q$  
P if and only if Q | The folder is blue if and only if it belongs to Tom.                     |

**Example 1:** Write the converse, inverse, and contra-positive of this statement:

   Conditional: If three points lie on the same line, then they are collinear.

   Converse: If three points are collinear, then they lie on the same line.
   Inverse: If three points are not on the same line, then they are not collinear.
   Contra-positive: If three points are not collinear, then they do not lie on the same line.
## Deductive Reasoning

<table>
<thead>
<tr>
<th>Term</th>
<th>Symbols</th>
<th>Example</th>
</tr>
</thead>
</table>
| **Law of Detachment** | 1. $P \rightarrow Q$
2. $P$
3. $Q$ | 1. If an angle is larger than $90^\circ$, then it is obtuse.  
2. $\angle ABC = 100^\circ$
3. $\angle ABC$ is obtuse |
| **Law of Syllogism**  | 1. $P \rightarrow Q$
2. $Q \rightarrow R$
3. *Therefore*, $P \rightarrow R$ | 1. If Joe is sick, he will miss work.  
2. If Joe misses work, he will not get paid.  
3. If Joe is sick, he will not get paid. |
| **Law of Contrapositive** | 1. $P \rightarrow Q$
2. $\sim Q$
3. *Then*, $\sim P$ | 1. If you study for the test, you will pass.  
2. You did not pass the test.  
3. You did not study. |
Answers to the \textit{TRY IT} problems:

\textbf{Solving Equations}

1. \( k = -19 \)
2. \( x = -15 \)
3. \( y = -28 \)
4. \( n = -\frac{1}{2} \)
5. \( k = 4 \)
6. \( g = -1 \)
7. \( m = -\frac{5}{3} \)
8. \( x = 2 \)

\textbf{Segments, Midpoint and Distance}

1. \( x = 6, PQ = 12, QR = 17 \)
2. \( \sqrt{104} \approx 10.20 \text{ or } 2\sqrt{26} \)
3. \( CD = 140 \)
4. \( Y (-4, -4) \)

\textbf{Writing Equations of Circles}

1. \( (1, -1), r = 7 \)
2. \( (-4, 0), r = \sqrt{8} \text{ or } 2\sqrt{2} \)
3. \( (x - 3)^2 + (y + 2)^2 = 121 \)
4. \( (x + 1)^2 + (y + 1)^2 = 1 \)
5. \( (x - 4)^2 + (y + 2)^2 = 16 \)
6. Sketch graph, see teacher.

\textbf{Angles}

1. \( \angle CFD = 39^\circ \)
2. \( x = 38 \)
3. No, because an acute angle measures less than \( 90^\circ \), therefore, even if you added two acute angles that are \( 89.99^\circ \) you still would not get \( 180^\circ \) which would make them supplementary.

\textbf{Parallel Lines and Transversals}

1. \( x = 7 \)
2. \( m = -5 \)
3. no
4. parallel
5. perpendicular