OVERVIEW

Geometry Content Review Notes are designed by the High School Mathematics Steering Committee as a resource for students and parents. Each nine weeks’ Standards of Learning (SOLs) have been identified and a detailed explanation of the specific SOL is provided. Specific notes have also been included in this document to assist students in understanding the concepts. Sample problems allow the students to see step-by-step models for solving various types of problems. A “” section has also been developed to provide students with the opportunity to solve similar problems and check their answers.

The document is a compilation of information found in the Virginia Department of Education (VDOE) Curriculum Framework, Enhanced Scope and Sequence, and Released Test items. In addition to VDOE information, Prentice Hall Textbook Series and resources have been used. Finally, information from various websites is included. The websites are listed with the information as it appears in the document.

Supplemental online information can be accessed by scanning QR codes throughout the document. These will take students to video tutorials and online resources. In addition, a self-assessment is available at the end of the document to allow students to check their readiness for the nine-weeks test.

The Geometry Blueprint Summary Table is listed below as a snapshot of the reporting categories, the number of questions per reporting category, and the corresponding SOLs.
# Geometry

**Test Blueprint Summary Table**

<table>
<thead>
<tr>
<th>Reporting Category</th>
<th>Geometry SOL</th>
<th>Number of Items</th>
</tr>
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<tbody>
<tr>
<td>Reasoning, Lines, and Transformations</td>
<td>G.1a-c</td>
<td>16</td>
</tr>
<tr>
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<td>G.2a-b</td>
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<tr>
<td>Triangles</td>
<td>G.5a-d</td>
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<td>G.6</td>
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<tr>
<td>Polygons, Circles, and Three-Dimensional Figures</td>
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<td></td>
<td>G.13</td>
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</tr>
<tr>
<td></td>
<td>G.14a-d</td>
<td></td>
</tr>
</tbody>
</table>

**Number of Operational Items**

45

**Number of Field-Test Items***

10

**Total Number of Items on Test**

55

*Field-test items are being tried out with students for potential use on subsequent tests and will not be used to compute students’ scores on the test.
Geometry Formula Sheet
2016 Mathematics Standards of Learning

Geometric Formulas

\[
\begin{align*}
A &= \frac{1}{2}bh \\
p &= 2l + 2w \\
A &= bh \\
A &= \frac{1}{2}h(b_1 + b_2)
\end{align*}
\]

Regular Hexagon

\[
\begin{align*}
A &= \frac{3\sqrt{3}}{2}s^2 \\
A &= \frac{1}{2}pa \\
C &= 2\pi r \\
C &= \pi d \\
V &= Bh \\
L.A. &= hp \\
S.A. &= hp + 2B \\
V &= lwh \\
S.A. &= 2lw + 2lh + 2wh
\end{align*}
\]

\[
\begin{align*}
V &= \pi r^2h \\
L.A. &= 2\pi rh \\
S.A. &= 2\pi r^2 + 2\pi rh \\
V &= \frac{4}{3}\pi r^3 \\
S.A. &= 4\pi r^2
\end{align*}
\]

\[
\begin{align*}
V &= \frac{1}{3}\pi r^2h \\
L.A. &= \pi rl \\
S.A. &= \pi r^2 + \pi rl \\
V &= \frac{1}{3}Bh \\
L.A. &= \frac{1}{2}lp \\
S.A. &= \frac{1}{2}lp + B
\end{align*}
\]
Geometry Formula Sheet
2016 Mathematics Standards of Learning

Geometric Formulas

\[ a^2 + b^2 = c^2 \]
\[ \sin \theta = \frac{o}{h} \]
\[ \cos \theta = \frac{a}{h} \]
\[ \tan \theta = \frac{o}{a} \]
\[ (x - h)^2 + (y - k)^2 = r^2 \]

Quadratic Formula:
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } ax^2 + bx + c = 0 \text{ and } a \neq 0 \]

<table>
<thead>
<tr>
<th>Geometric Symbols</th>
<th>Abbreviations</th>
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<tbody>
<tr>
<td><strong>Example</strong></td>
<td><strong>Meaning</strong></td>
</tr>
<tr>
<td>( m\angle A )</td>
<td>measure of angle ( A )</td>
</tr>
<tr>
<td>( AB )</td>
<td>length of line segment ( AB )</td>
</tr>
<tr>
<td>( \overline{AB} )</td>
<td>ray ( AB )</td>
</tr>
<tr>
<td>( \overline{AB} \parallel \overline{CD} )</td>
<td>Line ( AB ) is parallel to line ( CD ).</td>
</tr>
<tr>
<td>( \overline{AB} \perp \overline{CD} )</td>
<td>Line segment ( AB ) is perpendicular to line segment ( CD ).</td>
</tr>
<tr>
<td>( \angle A \cong \angle B )</td>
<td>Angle ( A ) is congruent to angle ( B ).</td>
</tr>
<tr>
<td>( \triangle ABC \sim \triangle DEF )</td>
<td>Triangle ( ABC ) is similar to triangle ( DEF ).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Abbreviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
</tr>
<tr>
<td>Area of Base</td>
</tr>
<tr>
<td>Circumference</td>
</tr>
<tr>
<td>Lateral Area</td>
</tr>
<tr>
<td>Perimeter</td>
</tr>
<tr>
<td>Surface Area</td>
</tr>
<tr>
<td>Volume</td>
</tr>
</tbody>
</table>

A | B | C | L.A. | p | S.A. | V |
Area and Perimeter

G.14 The student will apply the concepts of similarity to two- or three-dimensional geometric figures. This will include:

a) comparing ratios between lengths, perimeters, areas, and volumes of similar figures;

Many of the formulas for area and perimeter are included on your formula sheet.

Example 1: Find the perimeter of the rectangle pictured below.

The perimeter of a rectangle is found by adding the lengths of all of the sides together. We will use the distance formula to determine the measure of the length and the width.

\[
\overline{AB} = \sqrt{(3 - 7)^2 + (4 - 1)^2} \\
\overline{AC} = \sqrt{(-4)^2 + (3)^2} \\
\overline{AD} = \sqrt{(3 - (-3))^2 + (4 - (-4))^2} \\
\overline{AB} = \sqrt{6^2 + (8)^2} \\
\overline{AD} = \sqrt{36 + 64} = \sqrt{100} = 10
\]

\[p = 5 + 10 + 5 + 10 = 30\]

You can find the area of irregular figures by breaking it into pieces and then finding the area of each piece individually.

Example 2: Find the area of the shaded figure.

To find the area of the shaded region, we can first find the area of the whole rectangle, and then subtract the area of the removed triangle.

\[A_{rectangle} = lw = 22 \cdot 8 = 176\]

\[A_{triangle} = \frac{1}{2}bh\]

The height is the same as the width of the rectangle (8). The base is \(22 - 8 - 6 = 8\).

\[A_{triangle} = \frac{1}{2}bh = \frac{1}{2}(8)(8) = 32\]

\[A_{shaded} = 176 - 32 = 144\]
One of the area formulas that is not on your formula sheet is the formula for the area of a rhombus \( A_{rhombus} = \frac{1}{2} d_1 d_2 \) where \( d_1 \) and \( d_2 \) are the diagonals of the rhombus.

**Example 3:** Find the area of the rhombus pictured.

To find the area of the rhombus we need to know the lengths of each diagonal. Based on the properties of rhombi, we know that \( TX \) is 15 units, so one diagonal \( (RT) \) is 30 units.

We also know that all of the sides are congruent. We can then use right triangle QXR and the Pythagorean Theorem to find the length of the other diagonal.

\[
\begin{align*}
    a^2 + 15^2 &= 23^2 \\
    a^2 + 225 &= 529 \\
    a^2 &= 304 \\
    a &= \sqrt{304} \\
    QS &= 2\sqrt{304}
\end{align*}
\]

\[
A_{rhombus} = \frac{1}{2}(30)(2\sqrt{304}) \approx 523.07
\]

The area of a regular polygon is found by the formula \( A_{regular\ polygon} = \frac{1}{2} ap \) where \( a \) is the apothem (segment from center perpendicular to the side) and \( p \) is the perimeter.

You may have to use what you know about interior angles of regular polygons, trigonometry and special right triangles to find these measures.

**Example 4:** What is the area of a regular hexagon with a 4 in side?

We know that a regular hexagon with 4 in sides has a perimeter of 24 in. Now we need the apothem.

The apothem is \( AD \).

We can find the measure of \( \angle BAC \) by \( \frac{360}{6} = 60^\circ \).

This triangle is isosceles therefore \( \angle ACB \cong \angle ABC \) and they are also 60°.

That means that right triangle \( ADB \) is a 30 – 60 – 90 triangle with \( DB = 2 \text{ in} \).

Using what we know about 30-60-90 triangles, \( AD = 2\sqrt{2} \text{ in} \)

\[
A_{hexagon} = \frac{1}{2} ap = \frac{1}{2}(2\sqrt{2})(24) \approx 33.94 \text{ in}^2
\]
Example 5: What is the area of the regular pentagon shown below?

We are given the apothem, but we need to use trig to solve for the side length.
\[ \angle TPE = \frac{360}{5} = 72^\circ \] therefore \( \angle NPE = 36^\circ \). Now we can use trig to find \( EN \).

\[ \tan 36^\circ = \frac{EN}{10} \] therefore \( EN \approx 7.265 \) and \( ET \approx 14.53 \)

The perimeter of this pentagon is \( 14.53 \cdot 5 \approx 72.65 \)

\[ A_{pentagon} = \frac{1}{2}(10)(72.65) = 363.25 \]

Ratios can be used to compare the area and perimeter of similar figures.

If the scale factor of two similar figures is \( \frac{a}{b} \), then the ratio of their perimeters is \( \frac{a}{b} \), and the ratio of their areas is \( \frac{a^2}{b^2} \).

Example 6: These hexagons are similar figures. What is the area of the smaller figure, given that the area of the larger figure is \( 33.94 \text{ in}^2 \).

The scale factor for these similar figures is \( \frac{2}{4} \) or \( \frac{1}{2} \).

That means the ratio of their areas is \( \frac{1^2}{2^2} \) or \( \frac{1}{4} \).

Set up a proportion to solve for the area of the smaller figure.

\[ \frac{\text{ratio smaller}}{\text{ratio larger}} = \frac{\text{area smaller}}{\text{area larger}} \]

\[ \frac{1}{4} = \frac{x}{33.94} \]

\[ 4x = 33.94 \]

\[ x = 8.485 \text{ in}^2 \]

Scan this QR code to go to a video tutorial on Area and Perimeter.
**Area and Perimeter**

1. Find the area and perimeter of the figure below.

2. Find the shaded area of the regular hexagon.

3. What is the area of a regular octagon with side length 10 feet?

4. These triangles are similar. What is the scale factor?
**Volume and Surface Area**

**G.13** The student will use surface area and volume of three-dimensional objects to solve practical problems.

**G.14** The student will apply the concepts of similarity to two- or three-dimensional geometric figures. This will include:

- b) determining how changes in one or more dimensions of a figure affect area and/or volume of the figure;
- c) determining how changes in area and/or volume of a figure affect one or more dimensions of the figure;

<table>
<thead>
<tr>
<th>Volume and Surface Area Formulas</th>
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<tbody>
<tr>
<td><strong>Prism</strong></td>
</tr>
<tr>
<td>![Prism Diagram]</td>
</tr>
<tr>
<td>[ V = Bh ]</td>
</tr>
<tr>
<td>[ L.A. = hp ]</td>
</tr>
<tr>
<td>[ S.A. = hp + 2B ]</td>
</tr>
<tr>
<td>[ V = lwh ]</td>
</tr>
<tr>
<td>[ S.A. = 2lw + 2lh + 2wh ]</td>
</tr>
<tr>
<td>B: area of base</td>
</tr>
<tr>
<td>h: height</td>
</tr>
<tr>
<td>p: perimeter of base</td>
</tr>
<tr>
<td>l: length</td>
</tr>
<tr>
<td>w: width</td>
</tr>
<tr>
<td><strong>Cylinder</strong></td>
</tr>
<tr>
<td>![Cylinder Diagram]</td>
</tr>
<tr>
<td>[ V = \pi r^2 h ]</td>
</tr>
<tr>
<td>[ L.A. = 2\pi rh ]</td>
</tr>
<tr>
<td>[ S.A. = 2\pi r^2 + 2\pi rh ]</td>
</tr>
<tr>
<td>r: radius</td>
</tr>
<tr>
<td>h: height</td>
</tr>
<tr>
<td><strong>Cone</strong></td>
</tr>
<tr>
<td>![Cone Diagram]</td>
</tr>
<tr>
<td>[ V = \frac{1}{3} \pi r^2 h ]</td>
</tr>
<tr>
<td>[ L.A. = \pi rl ]</td>
</tr>
<tr>
<td>[ S.A. = \pi r^2 + \pi rl ]</td>
</tr>
<tr>
<td>r: radius</td>
</tr>
<tr>
<td>h: height</td>
</tr>
<tr>
<td>l: slant height</td>
</tr>
</tbody>
</table>
Pyramid

| V = \frac{1}{3} Bh | B: area of base 
| L.A. = \frac{1}{2} lp | h: height 
| S.A. = \frac{1}{2} lp + B | l: slant height 
| p: perimeter of base |

Sphere

| V = \frac{4}{3} \pi r^3 | r: radius 
| S.A. = 4\pi r^2 |

Scan this QR code to go to a video tutorial on Volume and Surface Area.

Example 1: Find the volume and surface area of the square base pyramid pictured.

I always start by writing the formulas that I need to use, and defining each of the variables.

Volume: \[ V = \frac{1}{3} Bh \]
\[ B = \text{area of base} = 440 \cdot 440 = 193600 \]
\[ h = \text{height} = 280 \]
\[ V = \frac{1}{3} (193600)(280) \approx 18,069,333.3 \]

Surface Area: \[ SA = \frac{1}{2} lp + B \]
\[ l = \text{slant height} = 356 \]
\[ p = \text{perimeter of base} = 440 \cdot 4 = 1760 \]
\[ B = \text{area of base} = 440 \cdot 440 = 193600 \]
\[ SA = \frac{1}{2} (356)(1760) + 193600 = 506,880 \]

Example 2: The volume of a cylinder is 504\pi. If the height is 14, solve for the radius.
\[ V = \pi r^2 h \]
\[ V = 504\pi \quad h = 14 \]
\[ 504\pi = \pi r^2 (14) \]
\[ \div 14\pi \quad \div 14\pi \]
\[ 36 = r^2 \]
\[ 6 = r \]
You can use ratios to compare the surface areas and volumes of similar solids.

Similar solids have the same shape and all corresponding dimensions are proportional.

If the scale factor of two similar figures is \( \frac{a}{b} \), then the ratio of their surface and lateral areas is \( \frac{a^2}{b^2} \) and the ratio of their volumes is \( \frac{a^3}{b^3} \).

Example 3: Are the two figures similar? If so give the scale factor.

Check the ratios of corresponding dimensions.

\[
\frac{7}{16} \neq \frac{10}{22}
\]

.4375 \( \neq \) .45

Therefore these figures are NOT similar!

Example 4: What is the scale factor of the similar pyramids pictured?

The ratio of their areas is \( \frac{32}{98} \) or \( \frac{16}{49} \).

The scale factor is the square root of the area ratio, therefore the scale factor is \( \frac{4}{7} \).
Volume and Surface Area

1. What is the volume and surface area of the cylinder.

2. The surface area of a prism is 166 in². If the length is 4 and the width is 5, what is the height?

3. Explain why all spheres are similar.

4. Find the volume of the smaller figure given that the surface area of the smaller figure is $8 \text{ cm}^2$, the volume of a larger similar figure is $500 \text{ cm}^3$ and the surface area of the larger figure is $50 \text{ cm}^2$.

5. If a cube with side length of 8 inches has its dimensions divided in half, what is the volume of the new cube?

6. Two plastic balls are sitting on a table. The first ball has a surface area of $196\pi \text{ cm}$. Find the surface area of the second ball if its radius is twice that of the first ball. (Express in terms of $\pi$.)
Transformations

G.3 The student will solve problems involving symmetry and transformation. This will include

c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.

<table>
<thead>
<tr>
<th>Transformations</th>
<th>Preimage</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Translation</strong></td>
<td>(x, y)</td>
<td>(x+6, y – 3)</td>
</tr>
<tr>
<td>All points of a figure travel the same distance and in the same direction. A translation is an isometry because the size doesn’t change.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| **Reflection** | r is the y-axis Because C is on r, it does not move. A and B are not on r, therefore r is the perpendicular bisector of AA’ and BB’.
| A reflection is the same as flipping a figure over an imaginary line r, called the line of reflection. A reflection across a line is an isometry. |

| **Rotation** | $R$ is $(-1, 1)$ Because C is on R, it does not move. A and B are not on R, therefore $\overline{RA} = \overline{RA'}$ and $m\angle ARA' = x$
| A rotation is the same as spinning a figure $x^\circ$ around an imaginary point R, called the center or rotation. A rotation about a point is an isometry. |
Dilation

A dilation is achieved by multiplying a figure by a scale factor $n$. If $n > 1$ the figure will grow. If $1 > n > 0$ the figure will shrink.

A dilation creates a similar figure but is not an isometry.

A dilation creates a similar figure but is not an isometry.

You can find the scale factor by

$$\frac{A'B'}{AB} = \frac{2}{6} = \frac{1}{3}$$

**Example 1:** List the coordinates of $\triangle A'B'C'$ which is $\triangle ABC$ reflected across the line $y = 2$.

Start by sketching in the line of reflection at $y = 2$. Then flip each point over that line. Points B and C are both 1 unit above the line of reflection which means that they will end up 1 unit below the line of reflection, but in the same position for the $x$-coordinate.

$B' (-3, 1)$ and $C' (3, 1)$

Point A is 4 units above the line of reflection. It will need to shift to 4 units below the line of reflection.

$A' (-3, -2)$. 
Example 2: Point O is the center of a regular octagon ABCDEFGH. What is the image of $\overline{CD}$ rotated $135^\circ$ clockwise about O.

Because this is a regular octagon it can be split into 8 congruent triangles. The measure of each central angle is $\frac{360}{8} = 45^\circ$.

A rotation of $135^\circ$ would move each vertex $\frac{135}{45} = 3$ vertices clockwise. This would put point C at point F, and point D at point G.

The image of $\overline{CD}$ rotated $135^\circ$ clockwise is $\overline{FG}$. 

Scan this QR code to go to a video tutorial on Transformations.
Transformations

1. What is the rule that describes the translation \( ABC \rightarrow A'B'C' \)

2. What are the coordinates of the vertices of \( \triangle XYZ \) for a dilation with center \((0, 0)\) and a scale factor of \(\frac{1}{3}\)?

3. What line of reflection maps point B at \((2, 1)\) to point B’ at \((2, 5)\)?
Answers to the problems:

**Area and Perimeter**

1. \( A = 200 \quad p \approx 61.54 \)
2. \( A = 63.64 \)
3. \( A = 60.35 \)
4. scale factor: \( \frac{2}{3} \)

**Volume and Surface Area**

1. \( V = 4200\pi \approx 13194.7 \)
   
   \( S.A. = 1040\pi \approx 3267.3 \)
2. \( h = 7 \text{ in} \)
3. There is only one dimension (radius) to consider.
4. \( V = 32 \text{ cm}^3 \)
5. \( 64 \text{ in.}^3 \)
6. \( 784\pi \text{ cm}^2 \).

**Transformations**

1. \( (x, y) \rightarrow (x + 9, y - 6) \)
2. \( X'(-1, 1), Y'(2, -1), Z'(-1, -1) \)
3. \( y = 3 \)