Geometry Content Review Notes are designed by the High School Mathematics Steering Committee as a resource for students and parents. Each nine weeks’ Standards of Learning (SOLs) have been identified and a detailed explanation of the specific SOL is provided. Specific notes have also been included in this document to assist students in understanding the concepts. Sample problems allow the students to see step-by-step models for solving various types of problems. A “TRY IT” section has also been developed to provide students with the opportunity to solve similar problems and check their answers.

The document is a compilation of information found in the Virginia Department of Education (VDOE) Curriculum Framework, Enhanced Scope and Sequence, and Released Test items. In addition to VDOE information, Prentice Hall Textbook Series and resources have been used. Finally, information from various websites is included. The websites are listed with the information as it appears in the document.

Supplemental online information can be accessed by scanning QR codes throughout the document. These will take students to video tutorials and online resources. In addition, a self-assessment is available at the end of the document to allow students to check their readiness for the nine-weeks test.

The Geometry Blueprint Summary Table is listed below as a snapshot of the reporting categories, the number of questions per reporting category, and the corresponding SOLs.

### Geometry Test Blueprint Summary Table

<table>
<thead>
<tr>
<th>Reporting Category</th>
<th>Geometry SOL</th>
<th>Number of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reasoning, Lines, and Transformations</td>
<td>G.1a-c G.2a-b G.3a-d G.4a-h</td>
<td>16</td>
</tr>
<tr>
<td>Triangles</td>
<td>G.5a-d G.6 G.7 G.8a-c</td>
<td>13</td>
</tr>
<tr>
<td>Polygons, Circles, and Three-Dimensional Figures</td>
<td>G.9 G.10a-c G.11a-d G.12 G.13 G.14a-d</td>
<td>16</td>
</tr>
<tr>
<td>Number of Operational Items</td>
<td></td>
<td>45</td>
</tr>
<tr>
<td>Number of Field-Test Items*</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Total Number of Items on Test</td>
<td></td>
<td>55</td>
</tr>
</tbody>
</table>

*Field-test items are being tried out with students for potential use on subsequent tests and will not be used to compute students’ scores on the test.
VDOE has not released final version of Spring 2019 formula sheet.
Geometry Formula Sheet
2009 Mathematics Standards of Learning

Geometric Formulas

\[ a^2 + b^2 = c^2 \]

\[ \sin \theta = \frac{y}{z} \]

\[ \cos \theta = \frac{x}{z} \]

\[ \tan \theta = \frac{y}{x} \]

\[ (x-h)^2 + (y-k)^2 = r^2 \]

\[ \pi \approx 3.14 \]

\[ \pi \approx \frac{22}{7} \]

Quadratic Formula:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{, where } ax^2 + bx + c = 0 \text{ and } a \neq 0 \]

Geometric Symbols

<table>
<thead>
<tr>
<th>Example</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m\angle A )</td>
<td>measure of angle ( A )</td>
</tr>
<tr>
<td>( AB )</td>
<td>length of line segment ( AB )</td>
</tr>
<tr>
<td>( \vec{AB} )</td>
<td>ray ( AB )</td>
</tr>
<tr>
<td>( \perp )</td>
<td>right angle</td>
</tr>
<tr>
<td>( AB \parallel CD )</td>
<td>Line ( AB ) is parallel to line ( CD ).</td>
</tr>
<tr>
<td>( AB \perp CD )</td>
<td>Line segment ( AB ) is perpendicular to line segment ( CD ).</td>
</tr>
<tr>
<td>( \angle A \cong \angle B )</td>
<td>Angle ( A ) is congruent to angle ( B ).</td>
</tr>
<tr>
<td>( \triangle ABC \sim \triangle DEF )</td>
<td>Triangle ( ABC ) is similar to triangle ( DEF ).</td>
</tr>
<tr>
<td>( \cong )</td>
<td>Similarly marked segments are congruent.</td>
</tr>
<tr>
<td>( \cong )</td>
<td>Similarly marked angles are congruent.</td>
</tr>
</tbody>
</table>
Algebra Review

Solving Equations

You will solve an equation to find all of the possible values for the variable. In order to solve an equation, you will need to isolate the variable by performing inverse operations (or ‘undoing’ what is done to the variable).

Any operation that you perform on one side of the equal sign MUST be performed on the other side as well. Drawing an arrow down from the equal sign may help remind you to do this.

Example 1: \[ m - 9 = -3 \]
\[ +9 \quad +9 \]
\[ m = 6 \]

Check your work by plugging your answer back in to the original problem.
\[ 6 - 9 = -3 \quad \checkmark \]

Example 2: \[ \frac{x + 4}{5} = -12 \]
\[ \cdot 5 \quad \cdot 5 \]
\[ x + 4 = -60 \]
\[ -4 \quad -4 \]
\[ x = -64 \]

Check your work by plugging your answer back in to the original problem.
\[ \frac{-64 + 4}{5} = \frac{-60}{5} = -12 \quad \checkmark \]
You may have to distribute a constant and combine like terms before solving an equation.

**Example 3:**

\[-4(g - 7) + 2g = -10\]

\[-4g + 28 + 2g = -10\]

\[-2g + 28 = -10\]

\[-28 \quad -28\]

\[-2g \quad = -38\]

\[\div (-2) \quad \div (-2)\]

\[g = 19\]

Check your work by plugging your answer back into the original problem.

\[-4 (19 - 7) + 2(19) = -10\]

\[-4(12) + 2(19) = -10\]

\[-48 + 38 = -10 \checkmark\]

If there are variables on both sides of the equation, you will need to move them all to the same side in the same way that you move numbers.

**Example 4:**

\[3p - 5 = 7(p - 3)\]

\[3p - 5 = 7p - 21\]

\[-3p \quad - 3p\]

\[-5 = 4p - 21\]

\[+21 \quad + 21\]

\[16 = 4p\]

\[\div 4 \quad \div 4\]

\[p = 4\]

Check your work by plugging your answer back into the original problem.

\[3(4) - 5 = 7(4) - 21\]

\[12 - 5 = 28 - 21\]

\[7 = 7 \checkmark\]
Example 5:

\[
\frac{x+10}{5x} = \frac{-1}{5}
\]

You can begin this problem by cross multiplying!

\[
5(x + 10) = -1(5x)
\]

\[
5x + 50 = -5x
\]

\[
+5x \quad +5x
\]

\[
10x + 50 = 0
\]

\[
-50 \quad -50
\]

\[
10x = -50
\]

\[
\div 10 \quad \div 10
\]

\[
x = -5
\]

Check your work by plugging your answer back in to the original problem.

\[
\frac{-5+10}{5(-5)} = \frac{-1}{5}
\]

\[
\frac{5}{-25} = \frac{-1}{5}
\]

\[
-\frac{1}{5} = -\frac{1}{5} \checkmark
\]

TRY IT:

Solving Equations

Solve each equation

1. \( k + 11 = -8 \)
2. \( 9 - 3x = 54 \)
3. \( -17 = \frac{y-6}{2} \)
4. \( 5(2n + 6) + 8 = 33 \)
5. \( 3 - (4k + 2) = -15 \)
6. \( 5g + 4 = -9g - 10 \)
7. \( -2(-4m - 1) + 3m = 4m - 8 + m \)
8. \( \frac{x-4}{2} = \frac{-2(3x-3)}{6} \)
### Points, Lines and Planes

**Definitions**

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Point</strong></td>
<td>A point is used to signify location only. A point has no length, width or depth. A point is represented by a dot and labeled with a capital letter.</td>
<td><img src="image" alt="Point Example" /></td>
</tr>
<tr>
<td><strong>Line</strong></td>
<td>A line has only one dimension: length. It continues indefinitely in both directions and is made up of infinitely many points. It can be labeled with a small script letter or by any two points on the line. (i.e. line v, PQ, QP)</td>
<td><img src="image" alt="Line Example" /></td>
</tr>
<tr>
<td><strong>Collinear Points</strong></td>
<td>Points that lie on the same line</td>
<td><img src="image" alt="Collinear Points Example" /></td>
</tr>
<tr>
<td><strong>Non-collinear Points</strong></td>
<td>Points that do NOT lie on the same line</td>
<td><img src="image" alt="Non-collinear Points Example" /></td>
</tr>
<tr>
<td><strong>Plane</strong></td>
<td>Thought of as a two dimensional (or flat) object that extends indefinitely in all directions. It can be labeled by a capital script letter or by three non-collinear points. (i.e. R, ABC, BCA, ACB, CBA)</td>
<td><img src="image" alt="Plane Example" /></td>
</tr>
<tr>
<td><strong>Co-Planar</strong></td>
<td>Points that lie on the same plane</td>
<td><img src="image" alt="Co-Planar Example" /></td>
</tr>
<tr>
<td><strong>Non Co-planar</strong></td>
<td>Points that do NOT lie on the same plane.</td>
<td><img src="image" alt="Non Co-planar Example" /></td>
</tr>
<tr>
<td><strong>Line Segment</strong></td>
<td>A portion of a line that is defined by two points. It has a finite length and no width. It is labeled by the two endpoints. (i.e. MN, NM)</td>
<td><img src="image" alt="Line Segment Example" /></td>
</tr>
</tbody>
</table>
Segments, Midpoint and Distance

G.3 The student will solve problems involving symmetry and transformation. This will include
a) investigating and using formulas for determining distance, midpoint, and slope;

The length of a line segment is determined by finding the distance between the two endpoints. Because a distance is always positive, to find the length you will take the absolute value of the difference between the two endpoints. \( MN = |x_1 - x_2| \).

Example 1: Find the length of segment AB.

\[
AB = |x_1 - x_2|
\]
\[
AB = |-2 - 3|
\]
\[
AB = |-5|
\]
\[
AB = 5
\]

Example 2: Find the distance between the table and the tree.

The distance between two ordered pairs can be determined using the distance formula.

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

You might find it helpful to label your points \((x_1, y_1)\) & \((x_2, y_2)\) before starting.
Example 3: Find the distance between X and Y if: X (3, -1) and Y (0, -4)

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ d = \sqrt{(0 - 3)^2 + (-4 - (-1))^2} \]

\[ d = \sqrt{(-3)^2 + (-3)^2} \]

\[ d = \sqrt{9 + 9} \]

\[ d = \sqrt{18} \approx 4.24 \]

Simplifying Radicals

In the last example, our answer was \( \sqrt{18} \). This radical can be simplified.

To simplify a radical, you will pull out any perfect square factors (i.e. 4, 9, 16, 25, etc.)

\[ \sqrt{18} = \sqrt{9 \cdot 2} \]

The square root of 9 is equal to 3, so you can pull the square root of 9 from underneath the radical sign to find the simplified answer \( 3\sqrt{2} \), which means 3 times the square root of 2. You can check this simplification in your calculator by verifying that \( \sqrt{18} = 3\sqrt{2} \).

Another way to simplify radicals, if you don’t know the factors of a number, is to create a factor tree and break the number down to its prime factors. When you have broken the number down to all of its prime factors you can pull out pairs of factors, which will multiply together to make perfect squares.

Example: Simplify \( \sqrt{128} \)

\[ \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 } \]

\[ 2 \cdot 2 \cdot 2 \sqrt{2} = 8\sqrt{2} \]
The midpoint of a line segment can be found using the formula \((\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})\).

**Example 4:** Find the midpoint between \((4, -6)\) and \((10, 2)\).

\[
\begin{align*}
\frac{4+10}{2}, \frac{-6+2}{2} \\
(7, -2)
\end{align*}
\]

**Example 5:** Given that \(M\) is the midpoint of \(PQ\), find \(x\) and determine the length of \(PQ\).

If \(M\) is the midpoint of \(PQ\), then \(PM = MQ \Rightarrow 5x - 4 = 2x + 8\)

\[
\begin{align*}
-2x & \quad -2x \\
3x - 4 & = 8 \\
+4 & +4 \\
3x & = 12 \\
\div3 & \div3 \\
x & = 4
\end{align*}
\]

If \(x = 4\), then \(PM = 5(4) - 4 = 20 - 4 = 16\) and \(MQ = 2(4) + 8 = 8 + 8 = 16\).

Therefore, \(PQ = PM + MQ\) or \(PQ = 16 + 16 = 32\)

Scan this QR code to go to a video tutorial on finding and using the midpoint.
G.4  The student will construct and justify the constructions of
a) line segment congruent to a given line segment;

<table>
<thead>
<tr>
<th>After doing this</th>
<th>Your work should look like this</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start with a line segment PQ that we will copy.</td>
<td><img src="image1" alt="Diagram" /></td>
</tr>
<tr>
<td>Step 1 Mark a point R that will be one endpoint of the new line segment.</td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td>Step 2 Set the compasses' point on the point P of the line segment to be copied.</td>
<td><img src="image3" alt="Diagram" /></td>
</tr>
<tr>
<td>Step 3 Adjust the compasses' width to the point Q. The compasses' width is now equal to the length of the line segment PQ.</td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
<tr>
<td>Step 4 Without changing the compasses' width, place the compasses' point on the the point R on the line you drew in step 1</td>
<td><img src="image5" alt="Diagram" /></td>
</tr>
<tr>
<td>Step 5</td>
<td>Without changing the compasses' width, draw an arc roughly where the other endpoint will be.</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Step 6</td>
<td>Pick a point $S$ on the arc that will be the other endpoint of the new line segment.</td>
</tr>
<tr>
<td>Step 7</td>
<td>Draw a line from $R$ to $S$.</td>
</tr>
<tr>
<td>Step 8</td>
<td>Done. The line segment $RS$ is equal in length (congruent to) the line segment $PQ$.</td>
</tr>
</tbody>
</table>

http://www.mathopenref.com/printcopysegment.html
TRY IT:  
Segments, Midpoint and Distance

1. Find $x$, $\overline{PQ}$, and $\overline{QR}$.

2. Find the distance between $(9, 2)$ and $(-1, 4)$.

3. Find $\overline{CD}$ if $M$ is the midpoint.

4. If the midpoint of line segment $\overline{XY}$ is $(0, -3)$ and $X$ is at $(4, -2)$, find the coordinates of $Y$.

5. Construct a segment congruent to segment $\overline{AB}$.
Writing Equations of Circles

G.12 The student will solve problems involving equations of circles.

Standard equation of a circle: \( x^2 + y^2 = r^2 \) where \( r \) is the radius of the circle, and the center of the circle is located at the origin \((0, 0)\).

When the center is not at the origin, you have to apply a shift to the x and y coordinates.

\[ (x - h)^2 + (y - k)^2 = r^2 \]

\( h \) is the shift in the x-coordinate, and \( k \) is the shift in the y-coordinate. Now, the center is located at \((h, k)\) and \( r \) is still the radius.

**Example 1:** Identify the center and radius of \( x^2 + (y + 3)^2 = 36 \)

The center is at \((0, -3)\), and the radius is 6

**Example 2:** Write the equation of a circle with center \((1, -5)\), \( r = \sqrt{5} \)

\[ (x - 1)^2 + (y + 5)^2 = 5 \]

**Example 3:** What is the standard equation of the circle shown below?

To write the standard equation of the circle we need to know the center of the circle, and the radius of the circle.

Center \((2, 0)\)

You can find the radius by counting how many units from the center to the edge. \( r = 5 \)

\[ (x - 2)^2 + (y - 0)^2 = 5^2 \]

\[ (x - 2)^2 + y^2 = 25 \]
Example 4: What is the standard equation of the circle shown below, given that the line shown is a diameter of the circle?

To write the standard equation of the circle we need to know the center of the circle, and the radius of the circle.

We can use the midpoint formula to find the center of the given diameter.

Center = \( \left( \frac{-6+2}{2}, \frac{3+0}{2} \right) = (-2, 1.5) \)

You can find the radius by using the distance formula. The radius is the distance from the center to either point on the edge of the circle.

\[
\begin{align*}
    r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} & \text{use points (2, 0) and (-2, 1.5)} \\
    r &= \sqrt{(-2 - 2)^2 + (1.5 - 0)^2} \\
    r &= \sqrt{(-4)^2 + (1.5)^2} \\
    r &= \sqrt{18.25}
\end{align*}
\]

\[
\begin{align*}
    (x - (-2))^2 + (y - 1.5)^2 &= (\sqrt{18.25})^2 \\
    (x + 2)^2 + (y - 1.5)^2 &= 18.25
\end{align*}
\]
Identify the center and radius of each circle.
3. \((x - 1)^2 + (y + 1)^2 = 49\)
4. \((x + 4)^2 + y^2 = 8\)

Write the equation of each circle.
5. center (3, -2), \(r = 11\)
6. center (-1, -1), \(d = 4\)
7. endpoints (5,-5) and (-1,-5)

2. Write the equation of the pictured circle.
**Angles**

**Definitions**

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ray</strong></td>
<td>Has one endpoint and extends indefinitely in the other direction. Named by endpoint and another point on the ray (endpoint must be first!) (i.e. ( XY ))</td>
<td><img src="image" alt="Ray" /></td>
</tr>
<tr>
<td><strong>Opposite Rays</strong></td>
<td>Two rays on the same line that point in opposite directions. (i.e. ( BA ) and ( BC ))</td>
<td><img src="image" alt="Opposite Rays" /></td>
</tr>
<tr>
<td><strong>Angle</strong></td>
<td>Formed by two non-collinear rays. Can be named by 3 points (vertex in the middle), vertex only (if only one angle is associated with that vertex), or with a number on the interior of the angle. (i.e. ( \angle DEF, \angle FED, \angle E, \angle 4 ))</td>
<td><img src="image" alt="Angle" /></td>
</tr>
</tbody>
</table>

**Angle Classifications**

<table>
<thead>
<tr>
<th>Acute</th>
<th>Right</th>
<th>Obtuse</th>
<th>Straight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measures less than 90°</td>
<td>Measures exactly 90°</td>
<td>Measures between 90° and 180°</td>
<td>Measures exactly 180°</td>
</tr>
</tbody>
</table>

**Angle Bisector**

A ray that divides an angle into two congruent halves (i.e. \( YA \) bisects \( XYZ \). Therefore \( \angle AYZ \cong \angle AYX \))

**Adjacent Angles**

Two angles that lie on the same plane, have a common vertex, and a common side. (i.e. \( \angle 2 \) and \( \angle 3 \) are adjacent)
<table>
<thead>
<tr>
<th>Geometry Term</th>
<th>Description</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vertical angles</strong></td>
<td>Non adjacent angles formed by two intersecting lines. Vertical angles are congruent. (i.e. $\angle 1$ and $\angle 3$ are vertical and $\angle 2$ and $\angle 4$ are vertical) Therefore $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$)</td>
<td></td>
</tr>
<tr>
<td><strong>Linear Pair</strong></td>
<td>Adjacent angles whose common side is a line. (i.e. $\angle 1$ and $\angle 2$ form a linear pair)</td>
<td></td>
</tr>
<tr>
<td><strong>Complementary Angles</strong></td>
<td>Pair of angles whose sum is 90° (i.e. $\angle GHK$ and $\angle KHJ$ are complementary)</td>
<td></td>
</tr>
<tr>
<td><strong>Supplementary Angles</strong></td>
<td>Pair of angles whose sum is 180° (i.e. $\angle 5$ and $\angle 6$ are supplementary angles)</td>
<td></td>
</tr>
<tr>
<td><strong>Perpendicular Lines</strong></td>
<td>Intersecting lines that form two right angles. (i.e. $\overline{AB} \perp \overline{XY}$)</td>
<td></td>
</tr>
</tbody>
</table>
Example 1: Find the measure of two supplementary angles if

\[ m\angle 1 = 7x \quad \text{and} \quad m\angle 2 = 3x - 10 \]

Because these angles are supplementary, we know that their sum equals 180°.

\[
7x + (3x - 10) = 180
\]

\[
10x - 10 = 180
\]

\[
+10 + 10
\]

\[
10x = 190
\]

\[
\div 10 \div 10
\]

\[
x = 19
\]

Once we know what x is equal to, we can solve for the measure of each angle!

\[
m\angle 1 = 7x = 7(19) = 133°
\]

\[
m\angle 2 = 3x - 10 = 3(19) - 10 = 57 - 10 = 47°
\]

Now you can verify that these are supplementary.

\[
133° + 47° = 180°
\]

Example 2: Find the value of x that will make \( \overrightarrow{BE} \perp \overrightarrow{AD} \)

If \( \overrightarrow{BE} \perp \overrightarrow{AD} \), then \( m\angle AFB = 90° \)

\[
6x + 18 = 90
\]

\[
-18 - 18
\]

\[
6x = 72
\]

\[
\div 6 \div 6
\]

\[
x = 12
\]
G.4 The student will construct and justify the constructions of
b) the perpendicular bisector of a line segment;

<table>
<thead>
<tr>
<th>After doing this</th>
<th>Your work should look like this</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Start with a line segment PQ.</strong></td>
<td><img src="image1" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>1</strong> Place the compasses on one end of the line segment.</td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>2</strong> Set the compasses' width to a approximately two thirds the line length. The actual width does not matter.</td>
<td><img src="image3" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>3</strong> Without changing the compasses' width, draw an arc above and below the line.</td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>4</td>
<td>Again without changing the compasses' width, place the compasses' point on one end of the line. Draw an arc above and below the line so that the arcs cross the first two.</td>
</tr>
<tr>
<td></td>
<td><img src="http://www.mathopenref.com/printbisectline.html" alt="Diagram" /></td>
</tr>
<tr>
<td>5</td>
<td>Using a straightedge, draw a line between the points where the arcs intersect.</td>
</tr>
<tr>
<td></td>
<td><img src="http://www.mathopenref.com/printbisectline.html" alt="Diagram" /></td>
</tr>
<tr>
<td>6</td>
<td>Done. This line is perpendicular to the first line and bisects it (cuts it at the exact midpoint of the line).</td>
</tr>
<tr>
<td></td>
<td><img src="http://www.mathopenref.com/printbisectline.html" alt="Diagram" /></td>
</tr>
</tbody>
</table>

http://www.mathopenref.com/printbisectline.html
The student will construct and justify the constructions of the bisector of a given angle;

<table>
<thead>
<tr>
<th>After doing this</th>
<th>Your work should look like this</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start with angle PQR that we will bisect.</td>
<td><img src="image1.png" alt="Diagram" /></td>
</tr>
</tbody>
</table>

1. Place the compasses' point on the angle's vertex Q.  
   ![Diagram](image2.png)

2. Adjust the compasses to a medium wide setting. The exact width is not important.  
   ![Diagram](image3.png)

3. Without changing the compasses' width, draw an arc across each leg of the angle.  
   ![Diagram](image4.png)
4. The compasses' width can be changed here if desired. Recommended: leave it the same.

5. Place the compasses on the point where one arc crosses a leg and draw an arc in the interior of the angle.

6. Without changing the compasses setting repeat for the other leg so that the two arcs cross.

7. Using a straightedge or ruler, draw a line from the vertex to the point where the arcs cross.

Done. This is the bisector of the angle \( \angle PQR \).

http://www.mathopenref.com/printbisectangle.html
The student will construct and justify the constructions of an angle congruent to a given angle;

<table>
<thead>
<tr>
<th>After doing this</th>
<th>Your work should look like this</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start with a angle BAC that we will copy.</td>
<td>![Diagram 1]</td>
</tr>
<tr>
<td>1. Make a point P that will be the vertex of the new angle.</td>
<td>![Diagram 2]</td>
</tr>
<tr>
<td>2. From P, draw a ray PQ. This will become one side of the new angle.</td>
<td>![Diagram 3]</td>
</tr>
<tr>
<td>• This ray can go off in any direction.</td>
<td></td>
</tr>
<tr>
<td>• It does not have to be parallel to anything else.</td>
<td></td>
</tr>
<tr>
<td>• It does not have to be the same length as AC or AB.</td>
<td></td>
</tr>
</tbody>
</table>
3. Place the compasses on point A, set to any convenient width.

4. Draw an arc across both sides of the angle, creating the points J and K as shown.

5. Without changing the compasses' width, place the compasses' point on P and draw a similar arc there, creating point M as shown.
6. Set the compasses on K and adjust its width to point J.

7. Without changing the compasses' width, move the compasses to M and draw an arc across the first one, creating point L where they cross.

8. Draw a ray PR from P through L and onwards a little further. The exact length is not important.

**Done.** The angle $\angle RPQ$ is congruent (equal in measure) to angle $\angle BAC$.
**Try It: Angles**

1. In the above diagram, given that \( \overline{BE} \perp \overline{AD} \) if \( \angle BFC = 51° \), what is the measure of \( \angle CFD \)?

2. Solve for \( x \): \( \angle 1 \) is complementary to \( \angle 2 \), \( m\angle 1 = 27 - 2x \) and \( m\angle 2 = 4x - 13 \)

3. Is it possible for two acute angles to be supplementary? Explain.

4. Construct the perpendicular bisector of segment \( \overline{AB} \).

![Diagram](image)

5. Construct an angle congruent to \( \angle A \).

![Diagram](image)

6. Construct the bisector of \( \angle B \).

![Diagram](image)
**Parallel Lines and Transversals**

**G.2** The student will use the relationships between angles formed by two lines intersected by a transversal to

a) prove two or more lines are parallel; and

b) solve problems, including practical problems, involving angles formed when parallel lines are intersected by a transversal.

**G.3** The student will solve problems involving symmetry and transformation. This will include

b) applying slope to verify and determine whether lines are parallel or perpendicular;

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parallel</strong></td>
<td>Coplanar lines that never intersect. These lines will have the same slope. Parallel lines are indicated with the ( \parallel ) symbol. (i.e. ( \overrightarrow{AB} \parallel \overrightarrow{CD} ))</td>
<td><img src="image1" alt="Parallel Lines" /></td>
</tr>
<tr>
<td><strong>Perpendicular</strong></td>
<td>Coplanar lines that intersect at a 90° angle. These lines will have slopes that are opposite reciprocals (i.e. ( \frac{2}{3} ) and ( -\frac{3}{2} )). Perpendicular lines are indicated with the ( \perp ) symbol. (i.e. ( \ell \perp m ))</td>
<td><img src="image2" alt="Perpendicular Lines" /></td>
</tr>
<tr>
<td><strong>Skew</strong></td>
<td>Describes lines or planes that are not coplanar, and do not intersect.</td>
<td><img src="image3" alt="Skew Lines" /></td>
</tr>
<tr>
<td><strong>Transversal</strong></td>
<td>A line that intersects two or more lines in a single plane. Two lines cut by a transversal do not always need to be parallel. Line ( \ell ) is a transversal through ( m ) and ( n ).</td>
<td><img src="image4" alt="Transversal" /></td>
</tr>
</tbody>
</table>
When a transversal intersects two lines, eight angles are formed. These angles have special properties when the two lines being intersected are parallel.

<table>
<thead>
<tr>
<th>Exterior Angles</th>
<th>∠’s a, b, g, h ∠’s 1, 2, 7, 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior Angles</td>
<td>∠’s c, d, e, f ∠’s 3, 4, 5, 6</td>
</tr>
<tr>
<td>Consecutive Interior Angles</td>
<td>∠c &amp; ∠e, ∠d &amp; ∠f ∠3 &amp; ∠5, ∠4 &amp; ∠6</td>
</tr>
<tr>
<td>Alternate Interior Angles</td>
<td>∠c &amp; ∠f, ∠d &amp; ∠e ∠3 &amp; ∠6, ∠4 &amp; ∠5</td>
</tr>
<tr>
<td>Alternate Exterior Angles</td>
<td>∠a &amp; ∠h, ∠b &amp; ∠g ∠1 &amp; ∠8, ∠2 &amp; ∠7</td>
</tr>
<tr>
<td>Corresponding Angles</td>
<td>∠a &amp; ∠e, ∠c &amp; ∠g, ∠b &amp; ∠f, ∠d &amp; ∠h ∠1 &amp; ∠5, ∠3 &amp; ∠7, ∠2 &amp; ∠6, ∠4 &amp; ∠8</td>
</tr>
</tbody>
</table>

If consecutive interior angles are supplementary, then the two lines are parallel.

If alternate interior angles are congruent, then the two lines are parallel.

If alternate exterior angles are congruent, then the two lines are parallel.

If corresponding angles are congruent, then the two lines are parallel.

Example 1: Find a value of x that will make a || b.

If a || b, then consecutive interior angles must be supplementary.

\[
2x + 1 + 135 = 180
\]

\[
2x + 136 = 180
\]

\[
-136
\]

\[
2x = 44
\]

\[
x = 22
\]
Slope is the change in the y coordinate, divided by the change in the x coordinate, (or rise over run). Slope is denoted by the letter m.  
\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

If two lines have the same slope, they are parallel. If two lines have slopes that are negative reciprocals, they are perpendicular.
Slopes can be positive, negative, zero, or undefined.

<table>
<thead>
<tr>
<th>Positive</th>
<th>Negative</th>
<th>Zero</th>
<th>Undefined</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Positive Slope" /></td>
<td><img src="image2.png" alt="Negative Slope" /></td>
<td><img src="image3.png" alt="Zero Slope" /></td>
<td><img src="image4.png" alt="Undefined Slope" /></td>
</tr>
</tbody>
</table>

**Example 2:** Find the slope of the line that goes through (0, 4) and (2, -3).

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 4}{2 - 0} = \frac{-7}{2} \]

**Example 3:** What would be the slope of a line perpendicular to the line in Example 2?

The slope of a perpendicular line will be the negative reciprocal of the slope of the original line. Therefore the slope of the perpendicular line is \[ m = \frac{2}{7} \].

You can determine the slope of a line when given the equation of the line by transforming the equation for y. Once y is on the side by itself, the equation is in slope-intercept form, and the slope is the coefficient of the x term.

\[ y = mx + b \]
Example 4: What is the slope of \(4x - 2y = 8\)?

Transform for \(y\):

\[
4x - 2y = 8 \\
-4x - 4x \\
-2y = -4x + 8 \\
\div (-2) \div (-2) \\
y = 2x - 4 \quad m = 2
\]

Example 5: What would be the slope of a line perpendicular to the line in Example 4?

The slope of a perpendicular line will be the negative reciprocal of the slope of the original line. Therefore the slope of the perpendicular line is \(m = -\frac{1}{2}\).

Scan this QR code to go to a video to learn more about slope and parallel and perpendicular lines.
G.4  The student will construct and justify the constructions of
g)  a line parallel to a given line through a point not on the given line;

<table>
<thead>
<tr>
<th>After doing this</th>
<th>Your work should look like this</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start with a line PQ and a point R off the line.</td>
<td><img src="image1" alt="Diagram" /></td>
</tr>
<tr>
<td>1. Draw a transverse line through R and across the line PQ at an angle, forming the point J where it intersects the line PQ. The exact angle is not important.</td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td>2. With the compasses' width set to about half the distance between R and J, place the point on J, and draw an arc across both lines.</td>
<td><img src="image3" alt="Diagram" /></td>
</tr>
<tr>
<td>3. Without adjusting the compasses' width, move the compasses to R and draw a similar arc to the one in step 2.</td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
</tbody>
</table>
4. Set compasses' width to the distance where the lower arc crosses the two lines.

5. Move the compasses to where the upper arc crosses the transverse line and draw an arc across the upper arc, forming point S.

6. Draw a straight line through points R and S.

Done. The line RS is parallel to the line PQ.

http://www.mathopenref.com/printparallel.html
The student will construct and justify the constructions of a perpendicular to a given line from a point not on the line;

<table>
<thead>
<tr>
<th>After doing this</th>
<th>Your work should look like this</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start with a line and point $R$ which is not on that line.</td>
<td><img src="image1.jpg" alt="Diagram" /></td>
</tr>
</tbody>
</table>

**Step 1** Place the compasses on the given external point $R$.  

**Step 2** Set the compasses' width to a approximately 50% more than the distance to the line. The exact width does not matter.  

**Step 3** Draw an arc across the line on each side of $R$, making sure not to adjust the compasses' width in between. Label these points $P$ and $Q$.  

![Diagram](image2.jpg)
<table>
<thead>
<tr>
<th>Step 4</th>
<th>At this point, you can adjust the compasses' width. Recommended: leave it as is. From each point P, Q, draw an arc below the line so that the arcs cross.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image1.png" alt="Diagram" /></td>
</tr>
<tr>
<td>Step 5</td>
<td>Place a straightedge between R and the point where the arcs intersect. Draw the perpendicular line from R to the line, or beyond if you wish.</td>
</tr>
<tr>
<td></td>
<td><img src="image2.png" alt="Diagram" /></td>
</tr>
<tr>
<td>Step 6</td>
<td>Done. This line is perpendicular to the first line and passes through the point R. It also bisects the segment PQ (divides it into two equal parts)</td>
</tr>
<tr>
<td></td>
<td><img src="image3.png" alt="Diagram" /></td>
</tr>
</tbody>
</table>

http://www.mathopenref.com/printperpextpoint.html
G.4 The student will construct and justify the constructions of a perpendicular to a given line at a given point on the line;

<table>
<thead>
<tr>
<th>After doing this</th>
<th>Your work should look like this</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start with a line and point K on that line.</td>
<td>![Diagram 1]</td>
</tr>
<tr>
<td>1 Set the compasses’ width to a medium setting. The actual width does not matter.</td>
<td>![Diagram 2]</td>
</tr>
<tr>
<td>2 Without changing the compasses’ width, mark a short arc on the line at each side of the point K, forming the points P, Q. These two points are thus the same distance from K.</td>
<td>![Diagram 3]</td>
</tr>
<tr>
<td>3 Increase the compasses to almost double the width (again the exact setting is not important).</td>
<td>![Diagram 4]</td>
</tr>
<tr>
<td>4 From P, mark off a short arc above K</td>
<td>![Diagram 5]</td>
</tr>
</tbody>
</table>
5. Without changing the compasses' width repeat from the point Q so that the two arcs cross each other, creating the point R.

6. Using the straight edge, draw a line from K to where the arcs cross.

7. Done. The line just drawn is a perpendicular to the line at K.

http://www.mathopenref.com/printperplinepoint.html
TRY IT: Parallel Lines and Transversals

1. What value of $x$ will make $a \parallel b$?

2. What is the slope of a line that would be perpendicular to the line that passes through $(4, -1)$ and $(9, 0)$?

3. Given $A (-1, 2) \& B (3, -5)$, and $C (7, 2) \& D (0, -2)$. Is $AB \parallel CD$?

4. Are these two lines parallel, perpendicular, or neither?
   \[ x = 3y - 8 \quad \text{and} \quad y = \frac{1}{3}x + 4 \]

5. Are these two lines parallel, perpendicular, or neither?
   \[ 4x - 3y = 6 \quad \text{and} \quad 4y = -3x + 9 \]

6. Construct a line parallel to line $\overrightarrow{YZ}$ through point $P$. 

![Diagram of parallel lines and transversals]
**Logic**

G.1 The student will use deductive reasoning to construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include:

- **a)** identifying the converse, inverse, and contrapositive of a conditional statement;
- **b)** translating a short verbal argument into symbolic form; and
- **c)** determining the validity of a logical argument.

<table>
<thead>
<tr>
<th>Term</th>
<th>Symbols</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditional Statement</td>
<td>( P \rightarrow Q )</td>
<td>If the folder is blue, then it belongs to Tom.</td>
</tr>
<tr>
<td></td>
<td>If P, then Q</td>
<td></td>
</tr>
<tr>
<td>Converse</td>
<td>( Q \rightarrow P )</td>
<td>If the folder belongs to Tom, then it is blue.</td>
</tr>
<tr>
<td></td>
<td>If Q, then P</td>
<td></td>
</tr>
<tr>
<td>Inverse</td>
<td>( \neg P \rightarrow \neg Q )</td>
<td>If the folder is not blue, then it does not belong to Tom.</td>
</tr>
<tr>
<td></td>
<td>If not P, then not Q</td>
<td></td>
</tr>
<tr>
<td>Contra-positive</td>
<td>( \neg Q \rightarrow \neg P )</td>
<td>If the folder does not belong to Tom, then the folder is not blue.</td>
</tr>
<tr>
<td></td>
<td>If not Q, then not P</td>
<td></td>
</tr>
<tr>
<td>Biconditional</td>
<td>( P \leftrightarrow Q )</td>
<td>The folder is blue if and only if it belongs to Tom.</td>
</tr>
<tr>
<td></td>
<td>P if and only if Q</td>
<td></td>
</tr>
</tbody>
</table>

**Example 1:** Write the converse, inverse, and contra-negative of this statement:

**Conditional:** If three points lie on the same line, then they are collinear.

**Converse:** If three points are collinear, then they lie on the same line.

**Inverse:** If three points are not on the same line, then they are not collinear.

**Contra-positive:** If three points are not collinear, then they do not lie on the same line.
### Deductive Reasoning

<table>
<thead>
<tr>
<th>Term</th>
<th>Symbols</th>
<th>Example</th>
</tr>
</thead>
</table>
| **Law of Detachment** | 1. $P \to Q$
2. $P$
3. $Q$ | 1. If an angle is larger than $90^\circ$, then it is obtuse.            |
|                       |         | 2. $\angle ABC = 100^\circ$                                            |
|                       |         | 3. $\angle ABC$ is obtuse                                              |
| **Law of Syllogism**  | 1. $P \to Q$
2. $Q \to R$
3. *Therefore*, $P \to R$ | 1. If Joe is sick, he will miss work.                                    |
|                       |         | 2. If Joe misses work, he will not get paid.                            |
|                       |         | 3. If Joe is sick, he will not get paid.                                |
| **Law of Contrapositive** | 1. $P \to Q$
2. $\sim Q$
3. *Then* $\sim P$ | 1. If you study for the test, you will pass.                            |
|                       |         | 2. You did not pass the test.                                           |
|                       |         | 3. You did not study.                                                   |
Answers to the problems:

**Solving Equations**

1. \( k = -19 \)
2. \( x = -15 \)
3. \( y = -28 \)
4. \( n = -\frac{1}{2} \)
5. \( k = 4 \)
6. \( g = -1 \)
7. \( m = -\frac{5}{3} \)
8. \( x = 2 \)

**Segments, Midpoint and Distance**

1. \( x = 6, \ PQ = 12, \ QR = 17 \)
2. \( \sqrt{104} \approx 10.20 \) or \( 2\sqrt{26} \)
3. \( CD = 140 \)
4. \( Y(-4, -4) \)
5. Measure for accuracy

**Writing Equations of Circles**

1. 

   ![Graph of a circle centered at (0, 0) with radius 4](image)

2. \( (x + 1)^2 + (y - 2)^2 = 16 \)

3. center: \((1, -1)\) radius: 7
4. center: \((-4, 0)\) radius: \(\sqrt{8}\) or \(2\sqrt{2}\)
5. \( (x - 3)^2 + (y + 2)^2 = 121 \)
6. \( (x + 1)^2 + (y + 1)^2 = 16 \)
7. \( (x - 2)^2 + (y + 5)^2 = 9 \)

**Angles**

1. \( \angle CFD = 39^\circ \)
2. \( x = 38 \)
3. No, because an acute angle measures less than 90°, therefore, even if you added two acute angles that are 89.99° you still would not get 180° which would make them supplementary.
4. Measure for accuracy
5. Measure for accuracy
6. Measure for accuracy

**Parallel Lines and Transversals**

1. \( x = 7 \)
2. \( m = -5 \)
3. no
4. parallel
5. perpendicular
6. Measure for accuracy