Geometry Content Review Notes are designed by the High School Mathematics Steering Committee as a resource for students and parents. Each nine weeks’ Standards of Learning (SOLs) have been identified and a detailed explanation of the specific SOL is provided. Specific notes have also been included in this document to assist students in understanding the concepts. Sample problems allow the students to see step-by-step models for solving various types of problems. A “**IT**” section has also been developed to provide students with the opportunity to solve similar problems and check their answers.

The document is a compilation of information found in the Virginia Department of Education (VDOE) Curriculum Framework, Enhanced Scope and Sequence, and Released Test items. In addition to VDOE information, Prentice Hall Textbook Series and resources have been used. Finally, information from various websites is included. The websites are listed with the information as it appears in the document.

Supplemental online information can be accessed by scanning QR codes throughout the document. These will take students to video tutorials and online resources. In addition, a self-assessment is available at the end of the document to allow students to check their readiness for the nine-weeks test.

The Geometry Blueprint Summary Table is listed below as a snapshot of the reporting categories, the number of questions per reporting category, and the corresponding SOLs.

<table>
<thead>
<tr>
<th>Reporting Category</th>
<th>Geometry SOL</th>
<th>Number of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reasoning, Lines, and Transformations</td>
<td>G.1a-d</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>G.2a-c</td>
<td></td>
</tr>
<tr>
<td></td>
<td>G.3a-d</td>
<td></td>
</tr>
<tr>
<td></td>
<td>G.4a-g</td>
<td></td>
</tr>
<tr>
<td>Triangles</td>
<td>G.5a-d</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>G.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>G.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>G.8</td>
<td></td>
</tr>
<tr>
<td>Polygons, Circles, and Three-Dimensional Figures</td>
<td>G.9</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>G.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>G.11a-c</td>
<td></td>
</tr>
<tr>
<td></td>
<td>G.12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>G.13</td>
<td></td>
</tr>
<tr>
<td></td>
<td>G.14a-d</td>
<td></td>
</tr>
<tr>
<td>Excluded from Testing</td>
<td>None</td>
<td>50</td>
</tr>
</tbody>
</table>

| Number of Operational Items                           | 50           |
| Number of Field-Test Items*                           | 10           |
| Total Number of Items on Test                         | 60           |

*Field-test items are being tried out with students for potential use on subsequent tests and will not be used to compute students’ scores on the test.
Geometry Formula Sheet
2016 Mathematics Standards of Learning

Geometric Formulas

\[ A = \frac{1}{2}bh \]
\[ p = 2l + 2w \]
\[ A = lw \]
\[ A = bh \]
\[ A = \frac{1}{2}h(b_1 + b_2) \]

Regular Hexagon

\[ A = \frac{3\sqrt{3}}{2} s^2 \]
\[ A = \frac{1}{2}pa \]
\[ C = 2\pi r \]
\[ C = \pi d \]
\[ A = \pi r^2 \]

V = Bh
L.A. = hp
S.A. = hp + 2B

V = lwh
S.A. = 2lw + 2lh + 2wh

V = \pi r^2h
L.A. = 2\pi rh
S.A. = 2\pi r^2 + 2\pi rh

V = \frac{4}{3}\pi r^3
S.A. = 4\pi r^2

V = \frac{1}{3}\pi r^2h
L.A. = \pi rl
S.A. = \pi r^2 + \pi rl

V = \frac{1}{3}Bh
L.A. = \frac{1}{2}lp
S.A. = \frac{1}{2}lp + B
Geometry Formula Sheet
2016 Mathematics Standards of Learning

Geometric Formulas

\[ a^2 + b^2 = c^2 \]

\[ \sin \theta = \frac{o}{h} \]
\[ \cos \theta = \frac{a}{h} \]
\[ \tan \theta = \frac{o}{a} \]

\[(x-h)^2 + (y-k)^2 = r^2\]

Quadratic Formula:
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } ax^2 + bx + c = 0 \text{ and } a \neq 0 \]

Geometric Symbols

<table>
<thead>
<tr>
<th>Example</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \angle A )</td>
<td>measure of angle ( A )</td>
</tr>
<tr>
<td>( AB )</td>
<td>length of line segment ( AB )</td>
</tr>
<tr>
<td>( \overrightarrow{AB} )</td>
<td>ray ( AB )</td>
</tr>
<tr>
<td>( AB \parallel CD )</td>
<td>Line ( AB ) is parallel to line ( CD ).</td>
</tr>
<tr>
<td>( AB \perp CD )</td>
<td>Line segment ( AB ) is perpendicular to line segment ( CD ).</td>
</tr>
<tr>
<td>( \angle A \cong \angle B )</td>
<td>Angle ( A ) is congruent to angle ( B ).</td>
</tr>
<tr>
<td>( \triangle ABC \sim \triangle DEF )</td>
<td>Triangle ( ABC ) is similar to triangle ( DEF ).</td>
</tr>
</tbody>
</table>

Abbreviations

<table>
<thead>
<tr>
<th>Area</th>
<th>( A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of Base</td>
<td>( B )</td>
</tr>
<tr>
<td>Circumference</td>
<td>( C )</td>
</tr>
<tr>
<td>Lateral Area</td>
<td>( L.A. )</td>
</tr>
<tr>
<td>Perimeter</td>
<td>( p )</td>
</tr>
<tr>
<td>Surface Area</td>
<td>( S.A. )</td>
</tr>
<tr>
<td>Volume</td>
<td>( V )</td>
</tr>
</tbody>
</table>

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**Triangle Congruency**

**G.6**  *The student, given information in the form of a figure or statement, will prove two triangles are congruent.*

Classifying Triangles

<table>
<thead>
<tr>
<th></th>
<th>Acute</th>
<th>Obtuse</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Acute Triangle" /></td>
<td><img src="image2" alt="Obtuse Triangle" /></td>
<td><img src="image3" alt="Right Triangle" /></td>
<td></td>
</tr>
<tr>
<td><strong>A triangle that has 3 acute angles.</strong></td>
<td><strong>A triangle that has one obtuse angle.</strong></td>
<td><strong>A triangle that has one right angle.</strong></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Equilateral</th>
<th>Equiangular</th>
<th>Isosceles</th>
<th>Scalene</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image4" alt="Equilateral Triangle" /></td>
<td><img src="image5" alt="Equiangular Triangle" /></td>
<td><img src="image6" alt="Isosceles Triangle" /></td>
<td><img src="image7" alt="Scalene Triangle" /></td>
<td></td>
</tr>
<tr>
<td><strong>A triangle whose sides are all congruent.</strong></td>
<td><strong>A triangle whose angles are all congruent.</strong></td>
<td><strong>A triangle with at least two congruent sides.</strong></td>
<td><strong>A triangle that has no congruent sides or angles.</strong></td>
<td></td>
</tr>
</tbody>
</table>
Triangle Angles Sum Theorem
The sum of the measures of the interior angles of a triangle is 180°.

\[ \angle A + \angle B + \angle C = 180° \]

Triangle Exterior Angle Theorem
The measure of each exterior angle of a triangle is equal to the sum of the measure of its two remote interior angles.

\[ \angle Z = \angle X + \angle Y \]

This makes sense because \( \angle W \) and \( \angle Z \) are supplementary, and the sum of \( \angle W, \angle X, \) and \( \angle Y \) would also be 180°.

Example 1: Solve for the missing angle.

\[ 31° + 82° + y = 180° \]
\[ 113° + y = 180° \]
\[ y = 67° \]

Example 2: Solve for \( x \).

\[ 25 + x + 15 = 3x - 10 \]
\[ x + 40 = 3x - 10 \]
\[ -x \quad -x \]
\[ 40 = 2x - 10 \]
\[ +10 +10 \]
\[ 50 = 2x \]
\[ x = 25 \]
Congruent Figures

Congruent Polygons have congruent corresponding parts. When naming congruent polygons, you must list the corresponding vertices in the same order.

Example 3: Given $\triangle LMN \cong \triangle PQR$, find $m\angle Q$.

Example 4: Given $\triangle AOB \cong \triangle YOZ$, find $x$.
### Triangle Congruence
You can prove that triangles are congruent without having to prove that all corresponding parts are congruent. We will learn 5 postulates that allow us to prove triangle congruence.

<table>
<thead>
<tr>
<th>Postulate</th>
<th>Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SSS</strong> (Side Side Side)</td>
<td>If three sides of one triangle are congruent to the three sides of another triangle, the two triangles are congruent.</td>
<td></td>
</tr>
<tr>
<td><strong>SAS</strong> (Side Angle Side)</td>
<td>If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.</td>
<td></td>
</tr>
<tr>
<td><strong>ASA</strong> (Angle Side Angle)</td>
<td>If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.</td>
<td></td>
</tr>
<tr>
<td><strong>AAS</strong> (Angle Angle Side)</td>
<td>If two angles and a non-included side of one triangle are congruent to two angles and a non-included side of another triangle, then the two triangles are congruent.</td>
<td></td>
</tr>
<tr>
<td><strong>HL</strong> (Hypotenuse Leg)</td>
<td>If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent.</td>
<td></td>
</tr>
</tbody>
</table>
are congruent.

**Example 5:** Given: $\overline{PS} \cong \overline{ST}$ and $\angle PSR \cong \angle RST$  **Prove:** $\overline{PR} \cong \overline{TR}$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{PS} \cong \overline{ST}$</td>
<td>Given</td>
</tr>
<tr>
<td>$\angle PSR \cong \angle RST$</td>
<td></td>
</tr>
<tr>
<td>$\overline{RS} \cong \overline{RS}$</td>
<td>Reflexive Property of</td>
</tr>
<tr>
<td></td>
<td>Congruence</td>
</tr>
<tr>
<td>$\Delta PRS \cong \Delta TRS$</td>
<td>SAS Postulate</td>
</tr>
<tr>
<td>$\overline{PR} \cong \overline{TR}$</td>
<td>Corresponding parts</td>
</tr>
<tr>
<td></td>
<td>of $\cong \triangle s$ are $\cong$.</td>
</tr>
</tbody>
</table>

Scan this QR code to go to a video tutorial on triangle congruence postulates and proofs.

**TRY IT:**

**Triangle Congruency**

1. The three angles of a triangle measure $58^\circ, (x + 41)^\circ, \text{and} \ (2x + 3)^\circ$. Solve for $x$.

2. Find the measure of $\angle ACD$. Given $m\angle BAC = 46^\circ$

3. Can you prove the triangles congruent? If so, which triangle congruence postulate would you use in each case?

   a. 

   ![Diagram](image1)

   b. 

   ![Diagram](image2)

   c. 

   ![Diagram](image3)

   d. 

   ![Diagram](image4)
Isosceles and Equilateral Triangles

An isosceles triangle is one that has two sides that are the same length. These sides are called legs. The third side is called the base.

The isosceles triangle theorem says that if two sides of a triangle are congruent, then the angles opposite of those sides are also congruent.

\[
\text{Given that } \overline{LN} \cong \overline{NM}, \text{ then } \angle L \cong \angle M
\]

The converse of this is also true!

\[
\text{Given that } \angle L \cong \angle M, \text{ then } \overline{LN} \cong \overline{NM}
\]

An equilateral triangle is one where all sides are congruent. As a corollary to the isosceles triangle theorem, if a triangle is equilateral then it is also equiangular.

\[
\text{Given that } \overline{AB} \cong \overline{BC} \cong \overline{AC}, \text{ then } \angle A \cong \angle B \cong \angle C
\]

The converse of this is also true!

\[
\text{Given that } \angle A \cong \angle B \cong \angle C, \text{ then } \overline{AB} \cong \overline{BC} \cong \overline{AC}
\]

**Example 6:** Find \( n \), given \( m\angle X = 18^\circ \)

Because this is an isosceles triangle, \( \angle Y \cong \angle Z \)

Therefore \( \angle Z = n \).

\[
\angle X + \angle Y + \angle Z = 180^\circ
\]

\[
18 + n + n = 180^\circ
\]

\[
2n + 18 = 180^\circ
\]

\[
-18 \quad -18
\]

\[
2n = 162
\]

\[
n = 81
\]
Occasionally you may be asked to prove congruence of two triangles that share common sides or angles, or that overlap.

It is often easier to separate the overlap, and to draw them as two separate triangles in order to prove congruence.

**Example 7: Given:** $\angle DAC \cong \angle CBD$ and $\angle ACD \cong \angle BDC$  **Prove:** $\Delta ACD \cong \Delta BDC$

![Triangles with angles and sides labeled]

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\angle DAC \cong \angle CBD$</td>
<td>Given</td>
</tr>
<tr>
<td>$\angle ACD \cong \angle BDC$</td>
<td>Reflexive Property of Congruence</td>
</tr>
<tr>
<td>$\overline{DC} \cong \overline{DC}$</td>
<td>AAS Postulate</td>
</tr>
<tr>
<td>$\Delta ACD \cong \Delta BDC$</td>
<td></td>
</tr>
</tbody>
</table>

**TRY IT:**

4. Explain why each interior angle of an equilateral triangle must measure $60^\circ$.

5. Given that $\overline{FJ}$ bisects $\angle HFG$ and $m\angle HFJ = 17^\circ$, find $m\angle G$.

6. What postulate could you use to prove $\Delta AED \cong \Delta BEC$?
**Triangle Similarity**

**G.5** The student, given information concerning the lengths of sides and/or measures of angles in triangles, will solve problems, including practical problems. This will include

a) order the sides by length, given the angle measures;
b) order the angles by degree measure, given the side lengths;
c) determine whether a triangle exists; and
d) determine the range in which the length of the third side must lie.

**G.7** The student, given information in the form of a figure or statement, will prove two triangles are similar.

**G.14** The student will apply the concepts of similarity to two- or three-dimensional geometric figures. This will include
d) solving problems, including practical problems, about similar geometric figures.

**Medians and Altitudes**

<table>
<thead>
<tr>
<th>Median</th>
<th>A segment that extends from a vertex of a triangle, and bisects the opposite side.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Median" /></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Centroid</th>
<th>The point where are three medians of a triangle intersect. This is also the center of gravity, or balance point.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image2.png" alt="Centroid" /></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Altitude</th>
<th>The segment that extends from one vertex of a triangle and is perpendicular to the opposite side of the triangle, or the line containing the opposite side. Altitudes can be inside, outside, or directly on a triangle side.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3.png" alt="Altitude" /></td>
<td></td>
</tr>
</tbody>
</table>
Orthocenter

The point where the three altitudes of a triangle intersect. An orthocenter can be inside, outside, or directly on a triangle.

Triangle Inequalities

If two sides of a triangle are not congruent, then the larger angle is opposite of the larger side.

Given $LN > MN > ML$
Then $\angle M > \angle L > \angle N$

The converse of this is also true. If two angles in a triangle are not congruent, then the larger side is opposite the larger angle.

Example 1: List the sides of the triangle in order from smallest to largest.

We know that the smallest side will be across from the smallest angle, and the largest side will be across from the largest angle. This means that $\overline{BC}$ is the smallest, and $\overline{AC}$ is the largest. Therefore listing from smallest to largest would be:

$\overline{BC}, \overline{AB}, \overline{AC}$

The triangle inequality theorem helps us to determine if 3 given lengths could form a triangle. The theorem states that in order for 3 sides to make a triangle, the sum of the lengths of the two shorter sides must be greater than the length of the longest side.
Example 2: Can the three lengths of 6 in, 7 in, and 8 in form a triangle?

Is $6 + 7 > 8$? Yes, $13 > 8$, therefore these 3 lengths can form a triangle.

Example 3: Can the three lengths of 4 mi, 8 mi, and 21 mi form a triangle?

Is $4 + 8 > 21$? No, $12$ is not greater than $21$, therefore these 3 lengths do NOT form a triangle.

Similar Polygons

Two polygons are similar if their corresponding angles are congruent, and the lengths of their corresponding sides are proportional.

$$ABCD \sim WXYZ$$

$$\angle A \cong \angle W \quad \angle B \cong \angle X$$
$$\angle C \cong \angle Y \quad \angle D \cong \angle Z$$

$$\frac{AB}{WX} = \frac{BC}{XY} = \frac{CD}{YZ} = \frac{DA}{ZW}$$

Example 4: Are the polygons similar? If they are, what is the scale factor?

Because the ratios of corresponding sides are congruent, these two figures are similar.

$$\triangle TUV \sim \triangle LMN$$ and the scale factor is $\frac{3}{1}$ or $3:1$. 
### Triangle Similarity

1. Can sides of length 7 ft, 11 ft, and 18 ft form a triangle? Explain.

2. Is $ABCD \sim FGHI$? If so, what is the scale factor?

#### Proving Triangle Similarity

<table>
<thead>
<tr>
<th>Similarity Type</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>$AA\sim$</strong></td>
<td>(Angle Angle Similarity)</td>
<td>If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\angle A \cong \angle D$ and $\angle B \cong \angle E$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Therefore $\triangle ABC \sim \triangle DEF$</td>
</tr>
<tr>
<td><strong>$SAS\sim$</strong></td>
<td>(Side Angle Side Similarity)</td>
<td>If two sides of one triangle are proportional to two sides of another triangle, and their included angles are congruent, then the triangles are similar.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{AC}{PR} = \frac{AB}{PQ}$ and $\angle A \cong \angle P$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Therefore $\triangle ABC \sim \triangle PQR$</td>
</tr>
<tr>
<td><strong>$SSS\sim$</strong></td>
<td>(Side Side Side Similarity)</td>
<td>If the corresponding sides of two triangles are proportional, then the triangles are similar.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{BC}{EF} = \frac{CD}{FG} = \frac{DB}{GE}$</td>
</tr>
</tbody>
</table>
Example 5: Are $\triangle ABC$ and $\triangle ADE$ similar? Explain why they are, or are not.

We can use SAS~, because each triangle has $\angle A$ as the included angle.

We just need to check to see if $\frac{AD}{AB} = \frac{AE}{AC}$.

$\frac{AD}{AB} = \frac{4}{6} = \frac{2}{3}$

$\frac{AE}{AC} = \frac{6}{10} = \frac{3}{5}$

Example 6: Given that $\triangle MTS \sim \triangle KQP$, solve for $y$.

Because the two triangles are similar, we know that the corresponding sides are proportional!

Set up a proportion to solve for $y$.

$\frac{ST}{PQ} = \frac{MT}{KQ}$

$\frac{3}{6} = \frac{4.2}{y}$

$3y = 25.2$

$y = 8.4$

Scan this QR code to go to a video tutorial on similar triangles.
Proportions in Triangles

<table>
<thead>
<tr>
<th>Side Splitter Theorem</th>
<th>If a line is parallel to one side of a triangle and intersects the other two sides, then it divides those sides up proportionally. (This is also true for three or more parallel lines intersecting any two transversals.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{DB}{BA} = \frac{EC}{CA} ]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Triangle-Angle-Bisector Theorem</th>
<th>If a ray bisects an angle of a triangle, then it divides the opposite side into two segments that are proportional to the other two sides of the triangle.</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{BA}{CA} = \frac{BP}{CP} ]</td>
<td></td>
</tr>
</tbody>
</table>

**Example 7:** Solve for \( x \).

Segment BD bisects \( \angle ABC \), therefore \( \frac{AD}{DC} = \frac{AB}{BC} \).

\[
\frac{3.5}{x} = \frac{5}{12}
\]

\[
5x = 42
\]

\[
x = 8.4
\]
Example 8: Given PQ \parallel TR, solve for x and y.

First, let’s use the side-splitter theorem to find x.

\[
\frac{SR}{RQ} = \frac{ST}{TP}
\]

\[
\frac{3}{9} = \frac{x}{15}
\]

\[
x = 45 / 9 = 5
\]

In order to find y, we first need to determine if \( \triangle PQS \sim \triangleTRS \).

These two triangles share \( \angle S \), therefore, if \( \frac{TS}{PS} \), the the triangles are similar by SAS~

\[
\frac{TS}{PS} = \frac{5}{20} = \frac{1}{4}
\]

\[
\frac{SR}{SQ} = \frac{3}{12} = \frac{1}{4}
\]

Because these ratios are equal, these two triangles are similar. This means that \( \frac{RT}{QP} = \frac{1}{4} \)

\[
\frac{y}{12} = \frac{1}{4}
\]

\[
4y = 12
\]

\[
y = 3
\]

Scan these QR codes to go to video tutorials on proportions in triangles.
**TRY IT:**

**Triangle Similarity**

3. Are the two triangles shown below similar? If so, write a similarity statement.

   a. 
   
   b. 

4. Explain why the triangles are similar, then find the length represented by y.

5. Solve for x.

6. Solve for x.
Right Triangles

G.8 The student will solve problems, including practical problems, involving right triangles. This will include applying
a) the Pythagorean Theorem and its converse;
b) properties of special right triangles; and
c) trigonometric ratios.

Pythagorean Theorem

The Pythagorean Theorem is an equation that compares the sides of a right triangle. It states that the sum of the squares of the two legs in a right triangle is equal to the square of the hypotenuse. Or more simply: \( a^2 + b^2 = c^2 \)

![Diagram of a right triangle with sides labeled a, b, and c.]

It is important to note, that the hypotenuse (c) is always across from the right angle, and is always the longest side of any right triangle.

Example 1: What is the length of the hypotenuse of a right triangle whose legs are 7.4 \text{ in} and 11 \text{ in}? Round your answer to the nearest hundredth.

\[
\begin{align*}
    a^2 + b^2 &= c^2 \\
    (7.4)^2 + (11)^2 &= c^2 \\
    54.76 + 121 &= c^2 \\
    175.76 &= c^2 \\
    \sqrt{175.76} &= \sqrt{c^2} \\
    13.26 \text{ in} &= c
\end{align*}
\]

Scan this QR code to go to a video tutorial on the Pythagorean Theorem.
Example 2: Solve for x. Round your answer to the nearest tenth.

\[ a^2 + b^2 = c^2 \]
\[ 9^2 + b^2 = 12.5^2 \]
\[ 81 + b^2 = 156.25 \]
\[ -81 \quad -81 \]
\[ b^2 = 75.25 \]
\[ \sqrt{b^2} = \sqrt{75.25} \]
\[ b = 8.7 \]

Sometimes you will be given 3 measurements of a triangle, and be asked to classify the type of triangle that those three side lengths would make.

If the three sides are integers and they form a right triangle, they are called Pythagorean Triples.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Equation</th>
<th>Example</th>
</tr>
</thead>
</table>
| Hypotenuse equal to sum of squares of legs | \( a^2 + b^2 = c^2 \) | \( 3^2 + 4^2 = 5^2 \)
| | | \( 9 + 16 = 25 \)
| | | \( 25 = 25 \) |
| Longest side greater than sum of squares of shorter sides | \( a^2 + b^2 < c^2 \) | \( 2^2 + 4^2 < 7^2 \)
| | | \( 4 + 16 < 49 \)
| | | \( 20 < 49 \) |
| Longer side less than sum of squares of shorter sides | \( a^2 + b^2 > c^2 \) | \( 6^2 + 7^2 > 8^2 \)
| | | \( 36 + 49 > 64 \)
| | | \( 85 > 64 \) |

Example 3: Are the three measurements 8, 12, 13 a Pythagorean Triple?

\[ 8^2 + 12^2 = 13^2 \quad ? \]
\[ 64 + 144 = 169 \quad ? \]
\[ 208 \neq 169 \]

Therefore this is not a Pythagorean Triple.
Example 4: What type of triangle is formed by sides of length 4\( ft \), 8\( ft \), and 11\( ft \)?

First use the triangle inequality to determine if the 3 sides do form a triangle.

\[
\text{Is } 4 + 8 > 11? \\
12 > 11
\]

So, now we know that the 3 sides can form a triangle. Now let’s determine what kind of triangle.

\[
\begin{align*}
4^2 + 8^2 &= 11^2 \\
16 + 64 &= 121 \\
80 &< 121
\end{align*}
\]

Therefore these three sides would form an obtuse triangle.

Special Right Triangles

There are two specific right triangles that have special properties:

45° – 45° – 90° and 30° – 60° – 90°

<table>
<thead>
<tr>
<th>45° – 45° – 90°</th>
<th>In a 45° – 45° – 90° triangle, the legs are congruent, and the hypotenuse is ( \sqrt{2} ) times the leg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>30° – 60° – 90°</td>
<td>In a 30° – 60° – 90° triangle, the hypotenuse is twice the length of the shorter leg. The longer leg is ( \sqrt{3} ) times the shorter leg.</td>
</tr>
</tbody>
</table>
Example 5: Find $x$ and $y$.

This is a 30-60-90 triangle, so we know that the shorter leg is $\frac{1}{2}$ the length of the hypotenuse.

Therefore $y = 5$.

The longer leg is $\sqrt{3}$ times the shorter leg.

Therefore $x = \sqrt{3} \cdot y$

$$x = 5\sqrt{3}$$

Example 6: Solve for $n$.

This is a 45-45-90 triangle, so we know that the hypotenuse is $\sqrt{2}$ times the length of the leg.

$$hyp = leg \sqrt{2}$$

$$10 = n\sqrt{2}$$

$$\div \sqrt{2} \quad \div \sqrt{2}$$

$$\frac{10}{\sqrt{2}} = n$$

Now we need to rationalize the denominator!

$$\frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

$$n = 5\sqrt{2}$$

Scan this QR code to go to a video tutorial on special right triangles.
Right Triangles (Part I)

1. A 24 ft ladder leans against a brick wall as shown in the picture below. If the base of the ladder must be at least 8 feet from the wall, can the ladder reach a window 20 ft above the ground?

2. The lengths of the sides of a triangle are given. Classify each as acute, obtuse, right, or not a triangle.
   a. 6, 8, 10
   b. 12, 13, 26
   c. 24.5, 30, 41.7

3. What is the length of the hypotenuse in an isosceles right triangle with legs 2ft?

4. Solve for u and w.

The trigonometric ratios can help you to determine missing information about RIGHT triangles. These ratios only work for RIGHT triangles. Calculator must be in DEGREE Mode.

<table>
<thead>
<tr>
<th>Sine</th>
<th>Cosine</th>
<th>Tangent</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin \angle = \frac{\text{Opposite}}{\text{Hypotenuse}} )</td>
<td>( \cos \angle = \frac{\text{Adjacent}}{\text{Hypotenuse}} )</td>
<td>( \tan \angle = \frac{\text{Opposite}}{\text{Adjacent}} )</td>
</tr>
</tbody>
</table>

Sin, Cos and Tan are used to find the lengths of sides of a right triangle. If you want to find a missing angle measure you will need to use an inverse \( (\sin^{-1}, \cos^{-1}, \text{or tan}^{-1}) \).
Example 1: What are the sine, cosine and tangent ratios for $\angle A$?

$$\sin \angle A = \frac{\text{opp}}{\text{hyp}} = \frac{5}{13}$$
$$\cos \angle A = \frac{\text{adj}}{\text{hyp}} = \frac{12}{13}$$
$$\tan \angle A = \frac{\text{opp}}{\text{adj}} = \frac{5}{12}$$

Example 2: Use your calculator to find the length of side $XY$.

We are given $\angle Z$ and the hypotenuse. $XY$ is opposite of $\angle Z$.
We will use the trig ratio that uses opposite and hypotenuse ($\sin$)

$$\sin \angle Z = \frac{\text{opp}}{\text{hyp}}$$
$$\sin 40^\circ = \frac{XY}{15}$$

$$XY = 15 \cdot \sin 40^\circ$$

$XY = 9.64$

These trig ratios can be used to find distances in word problems by creating right triangles with horizontal lines. The angles formed by these horizontal lines are often called angles of elevation and depression.
Example 3: A plane is one mile above sea level when it begins to climb at a constant angle of $2^\circ$ for the next 70 ground miles. How far above sea level is the plane after its climb?

*The first step here is to draw a picture to help you make sense of the problem.*
You need to solve for how much farther the plane is above sea level. We are given 2° and the side adjacent to it, and we need to find the side opposite. We will use the trig ratio that uses opposite and adjacent (tan).

\[ \tan \angle = \frac{opp}{adj} \]

\[ \tan 2 = \frac{x}{70} \]

\[ x = \tan 2 \cdot 70 \]

\[ x \approx 2.44 \text{ miles} \]

Because the plane was already 1 mile above sea level we need to add this to our value for x.

The plane is 3.44 miles above sea level.

TRY IT:

Right Triangles (Part II)

For problem 1 use Δ ABC at the right.

1. What are the sine, cosine, and tangent ratio for \( \angle B \)?

2. Find the measure of side ZY. Round to the nearest tenth.

3. A 6 foot tourist standing at the top of the Eifel Tower watches a ship pass under the Jena Bridge. If the angle of depression is 27° and the distance from the base of the Eifel Tower to the Jena Bridge is 504 feet, how tall is the Eifel Tower? Round to the nearest tenth. (Hint: You must calculate in the height of the tourist.)
Answers to the problems:

**Triangle Congruency**

1. \( x = 26 \)
2. \( 136^\circ \)
3. a. Yes, SAS
   b. Yes, SSS
   c. No
   d. Yes, HL
4. In an equilateral triangle, all sides are congruent, therefore all angles are congruent. The interior angles of a triangle sum to \( 180^\circ \), therefore the interior angles of an equilateral triangle are equal to \( \frac{180^\circ}{3} = 60^\circ \).
5. \( 73^\circ \)
6. SAS

**Right Triangles (Part I)**

1. Yes, the ladder can reach up to 22.6 ft.
2. a. Right Triangle
   b. Not a Triangle
   c. Obtuse Triangle
3. \( 2\sqrt{2} \) ft
4. \( u = 7 \quad w = 14 \)

**Right Triangles (Part II)**

1. \( \sin \angle B = \frac{\text{opp}}{\text{hyp}} = \frac{12}{13} \)
   \( \cos \angle B = \frac{\text{adj}}{\text{hyp}} = \frac{5}{13} \)
   \( \tan \angle B = \frac{\text{opp}}{\text{adj}} = \frac{12}{5} \)
2. 70.5
3. 983.2 feet

**Triangle Similarity**

1. No, because \( 7+11 \) is not greater than 18.
2. No
3. a. No
   b. \( \frac{AD}{AC} = \frac{AE}{AB} \), and the triangles share \( \angle A \)
   Therefore they are similar by SAS~
4. They are similar by AA~ \( y = 4.8 \)
5. \( x = 10 \)
6. \( x = 5 \)